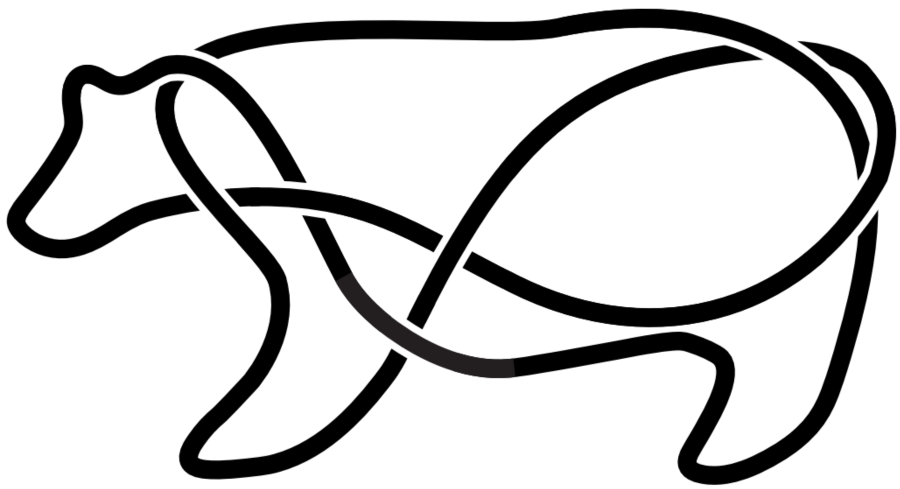
Berkeley Math Tournament 2025

Geometry Test



November 8, 2025

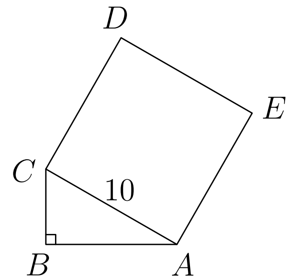
Time limit: 60 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

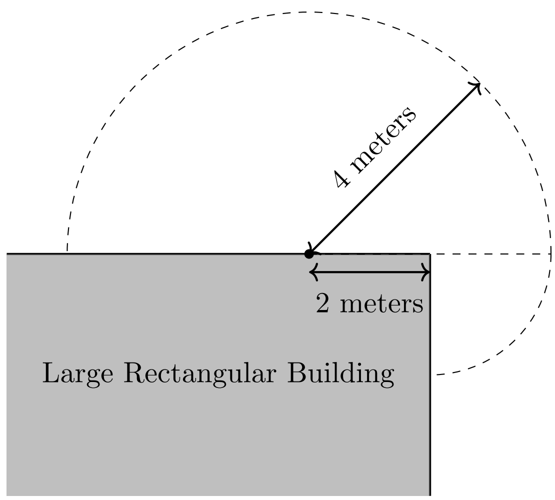
No calculators. Protractors, rulers, and compasses are permitted.

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.
- For fractions, both improper fractions and mixed numbers are acceptable.

1. Triangle $\triangle ABC$ has a right angle at B and all integer side lengths. Square $ACDE$ is constructed outside $\triangle ABC$. Given $AC = 10$, what is the perimeter of pentagon $ABCDE$?



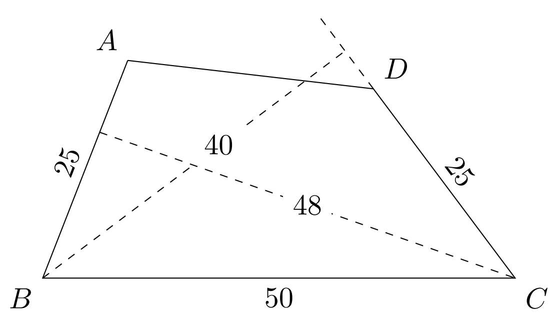
2. Stephanie uses a rope of length 4 meters to tie a sheep to a point that is 2 meters away from the corner of a large rectangular building. Given the sheep cannot enter the building, what is the area, in square meters, that the sheep can graze?



3. Square $ABCD$, regular pentagon $ABEFG$, and regular hexagon $ABHIJK$ all lie in the same plane so that the square lies inside the pentagon, and the pentagon lies inside the hexagon. Compute the degree measure of $\angle DGK$ which is less than $180°$.

4. Let triangle $\triangle ABC$ be equilateral. Points D and E lie on sides $\overline{AB}$ and $\overline{AC}$, respectively, such that $\overline{DE} \parallel \overline{BC}$ and $\overline{CD} \perp \overline{BE}$. Compute $\frac{AD}{AB}$.

5. Quadrilateral $ABCD$ has $AB = CD = 25$, $BC = 50$, and acute angles at vertices B and C. The distance from C to line $\overleftrightarrow{AB}$ is 48, and the distance from B to line $\overleftrightarrow{CD}$ is 40. Compute the area of quadrilateral $ABCD$.



6. Circles I and O have radii 1 and 8, respectively, and the distance between their centers is 4. Let $\mathcal{C}$ be the set of all circles simultaneously internally tangent to O and externally tangent to I. Compute the area enclosed by the curve formed from the centers of all circles in $\mathcal{C}$.

7. In three-dimensional coordinate space, a *lattice point* is a point whose coordinates are all integers. A triangular prism has all vertices at lattice points. Its triangular faces are both parallel to the xy-plane, and its rectangular faces are all parallel to the z-axis. The prism has 34 lattice points on its edges (including the vertices), 72 lattice points in the interior of its faces (not including the edges), and 126 lattice points in its interior (not including the faces). What is the volume of the prism?

8. In acute triangle $\triangle ABC$ with orthocenter H, let X be the unique point on $\overline{BC}$ for which angle $\angle AXH$ is maximized. Given $HX = 15, AX = 20$, and $BC = 30$, compute $AB + AC$.

9. Triangle $\triangle ABC$ has $AB = 4$, $AC = 5$, and $BC = 6$. Points E and F are the midpoints of sides $\overline{AB}$ and $\overline{AC}$, respectively. Let P be the foot of the altitude from A to side $\overline{BC}$, and let H be the midpoint of $\overline{AP}$. The circumcircles of triangles $\triangle HEB$ and $\triangle HFC$ intersect at $K \neq H$. Compute AK^2.

10. Triangle $\triangle ABC$ has $AB = 3 + \sqrt{2}$ and $AC = 1 + \sqrt{2}$. Point D lies on side $\overline{BC}$ such that $AD = 2 - \sqrt{2}$. The internal angle bisector of $\angle BAD$ intersects the circumcircle of triangle $\triangle BAD$ at $X \neq A$, and the internal angle bisector of $\angle CAD$ intersects the circumcircle of triangle $\triangle CAD$ at $Y \neq A$. Given $\overline{XY} \parallel \overline{BC}$, compute XY.