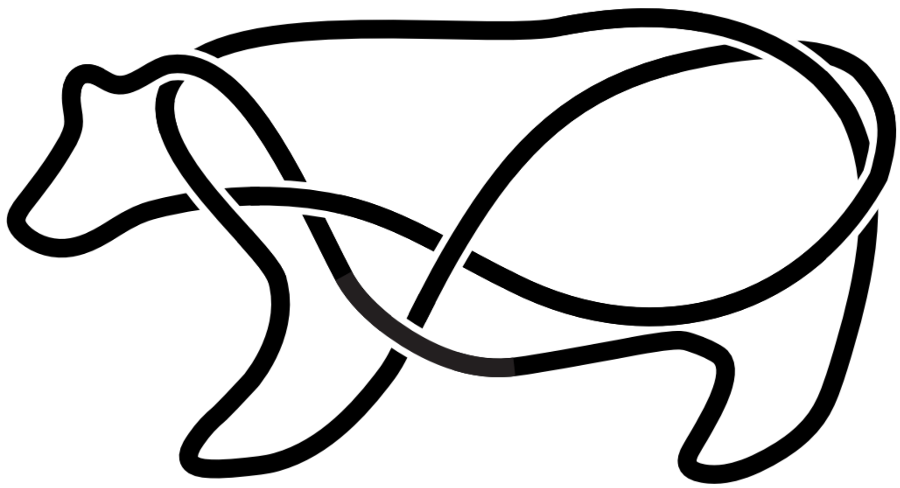
Berkeley Math Tournament 2025

General Test



November 8, 2025

Time limit: 90 minutes.

Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators. Protractors, rulers, and compasses are permitted.

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2} + \frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.
- For fractions, both improper fractions and mixed numbers are acceptable.

1. Compute
$$5^2 + 8^2 - 2 \cdot 5 \cdot 8 + 58.$$

2. An equilateral triangle and a square have the same perimeter. Given that the sides of the triangle each have length 8, compute the area of the square.

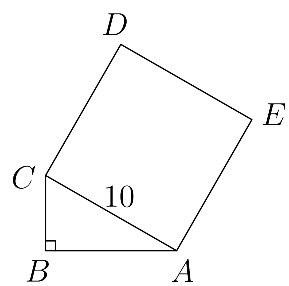
3. Find the smallest two-digit positive integer whose digits sum to a multiple of 6 and whose ones digit is twice its tens digit.

4. Ylann has four shirts (red, orange, yellow, and green) and two pairs of pants (black and white). An outfit consists of one shirt and one pair of pants. If Ylann refuses to wear a red shirt with white pants, and he also refuses to wear a yellow shirt with black pants, how many acceptable outfits can he make?

5. Oliver writes the number 1 on the board and then makes a series of moves. In each move, Oliver either multiplies the number on the board by 3 or increases it by 20. What is the minimum number of moves Oliver needs to change the number on the board to 67?

6. Jonathan and Carlyana race around a circular track with circumference 40 meters. They run clockwise at constant speeds: Jonathan runs at 5 meters per second, and Carlyana runs at 10 meters per second. They clap whenever they are at the same position on the track at the same time. If the first clap occurs at the start of the race, how many seconds after the start of the race does the second clap occur?

7. Triangle $\triangle ABC$ has a right angle at B and all integer side lengths. Square $ACDE$ is constructed outside $\triangle ABC$. Given $AC = 10$, what is the perimeter of pentagon $ABCDE$?



8. Jordan has two white socks and two black socks in their sock drawer. If they pull out two socks uniformly at random without replacement, what is the probability that they are the same color?

9. A hat initially contains five slips labeled with the numbers 1, 2, 3, 4, and 5 (one slip per number). Every minute, Jessica adds a new slip labeled with the number 20 to the hat. How many slips are in the hat when the average of the numbers in the hat is 5 times the initial average?

10. Let A, B, and C be digits such that $\underline{A}\,\underline{B}\,\underline{C} \times \underline{A}\,\underline{C} = 2025$. Find the three-digit number $\underline{A}\,\underline{B}\,\underline{C}$.

11. Find the real number x satisfying $\sqrt{x+3} - \sqrt{x-1} = 1$.

12. Find the positive integer n such that $n + 10$ is both a multiple of n and a factor of $n + 100$.

13. An arithmetic sequence is a sequence of numbers where the difference between any two consecutive terms is the same. Harsh writes an arithmetic sequence where the ratio of the second term to the first term is 4, and the sum of the third and fourth terms is 34. Compute the second term of Harsh's sequence.

14. A square of area A is divided using 2 lines into 4 rectangles (which may be squares) with all integer side lengths. Two of the rectangles have areas of 4 and 9, respectively. Find the sum of both distinct possible values of A.

15. Ariel places 4 green stickers, 4 orange stickers, and 4 purple stickers on the faces of two fair 6-sided dice such that face has exactly one sticker, and each die has at least one sticker of every color. If these dice are rolled, the probability that two green faces are rolled is $\frac{1}{12}$, and the probability that two orange faces are rolled is $\frac{1}{12}$. What is the probability that two purple faces are rolled?

16. Square $ABCD$, regular pentagon $ABEFG$, and regular hexagon $ABHIJK$ all lie in the same plane so that the square lies inside the pentagon, and the pentagon lies inside the hexagon. Compute the degree measure of $\angle DGK$ which is less than $180°$.

17. In how many ways can ten identical white blocks be placed into a red box, a yellow box, a green box, and a blue box such that no box is empty, and at least two boxes contain the same number of blocks?

18. Benji writes down the decimal representation of a rational number greater than 10. He observes that if he replaces the tens digit with 2, the number decreases by 70%. If he instead replaces the tens digit with 8, the number increases by 14%. By what percent would the number decrease if he replaced the tens digit with 5?

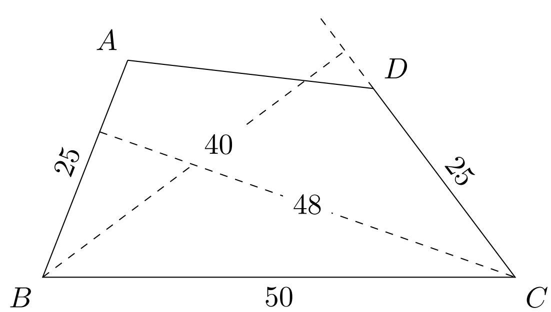
19. A rectangle is reflected over one of its diagonals, resulting in a second rectangle. Given that the region inside both of these rectangles is a rhombus with perimeter 52 and area 120, what is the perimeter of the original rectangle?

20. Let triangle $\triangle ABC$ be equilateral. Points D and E lie on sides $\overline{AB}$ and $\overline{AC}$, respectively, such that $\overline{DE} \parallel \overline{BC}$ and $\overline{CD} \perp \overline{BE}$. Compute $\frac{AD}{AB}$.

21. A *substring* of a positive integer n is a positive integer whose digits appear consecutively in the digits of n. For example, 10, $00 = 0$, and $02 = 2$ are two-digit substrings of 1002, but 12 and $01 = 1$ are not. An integer n is chosen uniformly at random from the integers between 1000 and 9999 inclusive. Compute the probability that all two-digit substrings of n are divisible by three.

22. Over all triples of positive integers (a, b, c) satisfying the equation
$$\gcd(\operatorname{lcm}(a, b), \operatorname{lcm}(a, c), \operatorname{lcm}(b, c)) = 2025,$$
compute the smallest possible value of $a + b + c$.

23. Quadrilateral $ABCD$ has $AB = CD = 25$, $BC = 50$, and acute angles at vertices B and C. The distance from C to line $\overleftrightarrow{AB}$ is 48, and the distance from B to line $\overleftrightarrow{CD}$ is 40. Compute the area of quadrilateral $ABCD$.



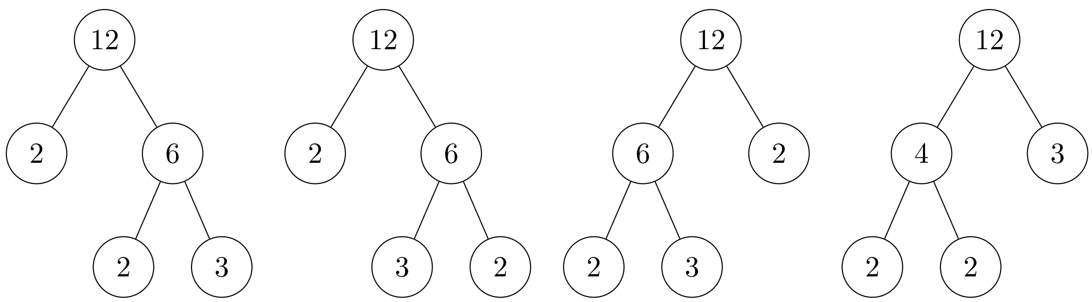
24. Compute the sum of all distinct real numbers x satisfying $|x| \leq \frac{1}{2}$ and

$$\sin\left(\log_{10}\left(\frac{1}{x}+\frac{1}{x^2}\right)\right) = 0.$$

Note that the argument of the sine function is assumed to be in radians.

25. A *factor tree* for a composite number n is a binary tree with root n and two children, a and b, such that a and b are proper divisors of n with $ab = n$, and whose subtrees rooted at a and b are themselves factor trees for a and b, respectively. A factor tree for a prime number is the single node containing the number itself.

Four distinct factor trees for 12 are shown below.



How many distinct factor trees are there for 120?