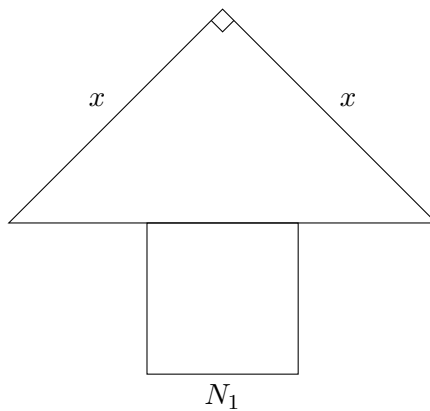


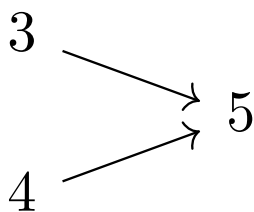
Set 1

$$1 \longrightarrow 2$$

1. Arthur has 25 apples and Ben has 5 apples. Arthur gives Ben some of his apples. If Arthur ends up with twice as many apples as Ben, how many apples did Arthur give Ben?
2. Let N_1 be the answer to Problem 1.

An isosceles right triangle with legs of length x is drawn on top of a square with side length N_1 , as shown below. If the area of the entire figure is 3 times the area of the square, find the value of x .



Set 2

3. Calculate

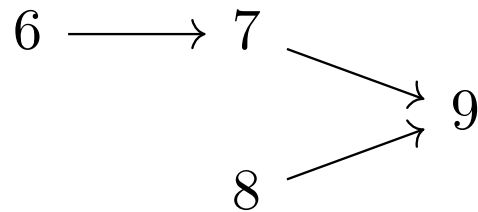
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6}.$$

4. There are 16 students in Mr. Sy's class. On Monday, some of his students took a test and got an average score of 50. On Tuesday, the remaining students were tested and got an average score of 90. If the average score of the whole class was 65, how many students took the test on Tuesday?

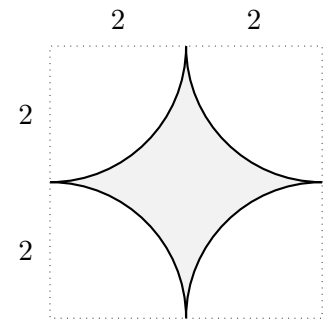
5. Let N_3 be the answer to Problem 3 and N_4 be the answer to Problem 4.

How many sequences of $N_3 \cdot N_4$ coin flips contain at least two heads in a row **and** at least two tails in a row?

Set 3



6. A star (the shaded region of the figure to the right) is created by cutting out four quarter circles of radius 2 from a square of side length 4, where the quarter circles are centered at the corners of the square. Let A and P be the area and perimeter of the star, respectively. What is $A + P$?



7. Let N_6 be **half** of the answer to Problem 6.

Gabriel is making a list of positive integers. He starts his list with the number N_6 . To find each new number for his list, he follows these steps using the previous number:

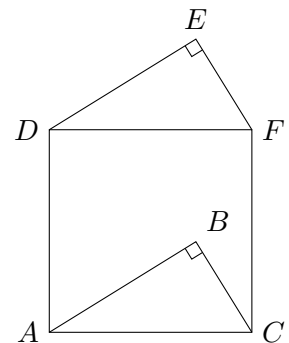
1. Find all the factors of that number (including 1 and the number itself).
2. Sum all those factors together and add 1 more to that sum.
3. The result of step 2 is the next number in the list.

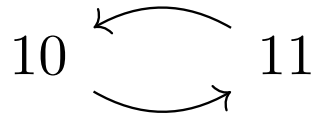
Gabriel stops finding new numbers once his list contains a number greater than or equal to 2^{N_6} . How many numbers are in his list?

8. Alice has a bowl, 20 grams of water, and 15 grams of flour. She mixes some of the flour and water together in the bowl. After this, there are twice as many remaining grams of water as remaining grams of flour. She decides to pour half of that remaining flour into the bowl as well. If there are now exactly 15 grams of mixture in the bowl, how many grams of water are in the bowl?

9. Let N_7 be the answer to Problem 7 and N_8 be the answer to Problem 8.

Square $ADFC$ has side length N_7 . Points B and E are chosen with B inside square $ADFC$ and E outside square $ADFC$ such that right triangles $\triangle ABC$ and $\triangle DEF$ are congruent with $\angle B = \angle E = 90^\circ$ and $BC = EF = N_8$. Find the area of quadrilateral $DEBA$.



Set 4

10. Let N_{11} be the answer to Problem 11.

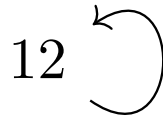
Alice has twenty cards, each labeled with a single number:

- Ten cards have a 1 on them,
- Nine cards have a 2 on them, and
- One card has N_{11} on it.

She groups up the cards into ten pairs and then computes the product of the numbers in each pair. What is the maximum possible value of the sum of the ten products she computes?

11. Let N_{10} be the answer to Problem 10.

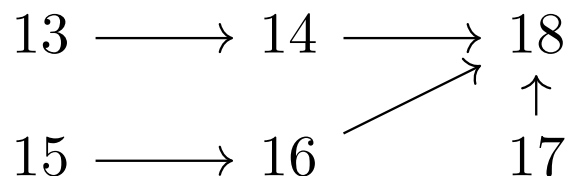
There are l distinct parallel lines on a plane. Three new lines are drawn, each one distinct from all previous lines, such that the total number of intersection points among all $l + 3$ lines is maximized. If there are N_{10} intersection points in total, find l .

Set 5

12. Let N_{12} be the answer to this problem.

How many integers between N_{12} and $3600 - N_{12}$, inclusive, are divisible by $3N_{12} + 3$ but **not** by $2N_{12} + 2$?

Set 6



13. How many ways can five people of distinct heights stand in a line such that no three consecutive people have heights in increasing or decreasing order, and exactly two people stand between the shortest person and the tallest person?

14. Let N_{13} be the answer to Problem 13.

Positive integers a and b , neither of which divides the other, are chosen such that

$$\text{lcm}(a, b) > 13N_{13},$$

$$\text{gcd}(a, b) > N_{13},$$

and $a + b$ is minimized. Find $\text{lcm}(a, b)$.

15. Rectangle $ABCD$ has side lengths $AB = 20$ and $BC = 26$. Points M and T lie on line segments \overline{AB} and \overline{AD} , respectively, such that $AT = 6$, $\angle ATM = \angle MTC$, and $\angle BMT = x^\circ$. Find x .

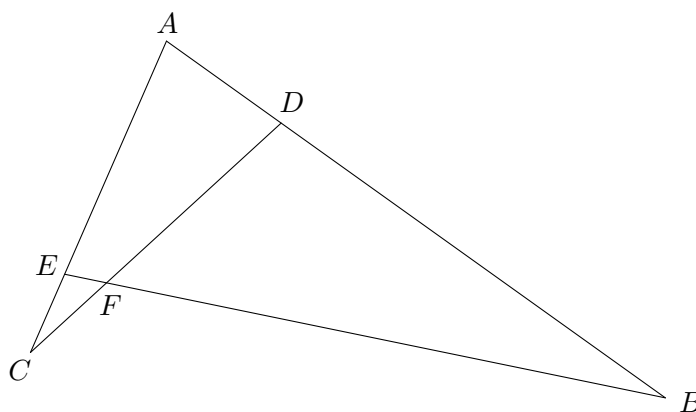
16. Let N_{15} be the answer to Problem 15.

Positive integers x and y satisfy

$$x^2 + x = y^2 + y + \frac{16}{35}N_{15}.$$

Find the sum of all possible values of x .

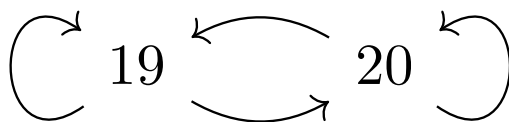
17. In the figure below, points D and E lie on segments \overline{AB} and \overline{AC} , respectively, such that $AC = CD = 8$ and $AB = BE = 10$. If $AD = 4$ and segments \overline{BE} and \overline{CD} intersect at F , find the area of non-convex quadrilateral $ACFB$.



18. Let N_{14} be the **sum of the digits** of the answer to Problem 14, N_{16} be the answer to Problem 16, and N_{17} be the answer to Problem 17.

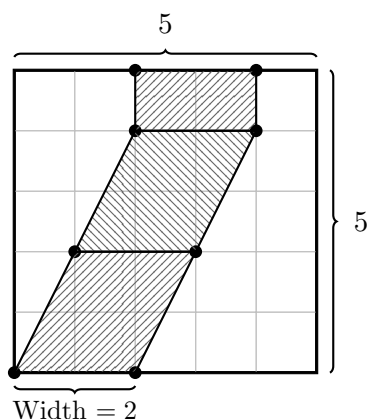
In how many ways can the four variables a, b, c, d be assigned not necessarily distinct values from the set $\{N_{14}, N_{16}, N_{17}\}$ such that $a^2 + bc$, $ab + bd$, $ac + cd$, and $bc + d^2$ all have distinct values?

Set 7

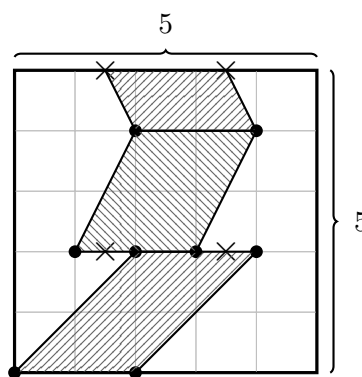


19. Let N_{19} be the answer to this problem and N_{20} be the answer to Problem 20.

There are M ways to draw 3 parallelograms of positive area and width N_{20} stacked from base-to-base connecting the bottom to the top of an $N_{19} \times N_{19}$ grid of unit squares so that the vertices of the parallelograms lie on intersecting grid lines. Given M is a multiple of $2^{N_{20}}$, find N_{19} .



Example using
 $N_{19} = 5, N_{20} = 2$



Non-example

20. Let N_{19} be the answer to Problem 19 and N_{20} be the answer to this problem.

A middle school surveyed 101 students and found that

- 48 students like math,
- $2N_{20}$ students like English,
- $2N_{19}$ students like history,
- N_{19} students like (both) math and English, and
- 1 student dislikes all three subjects.

How many students like (both) math and history?

