Berkeley mini Math Tournament 2025

Team Round

April 12, 2025

Time limit: 60 minutes.

Instructions: For this test, you will work in teams of up to five to solve 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.

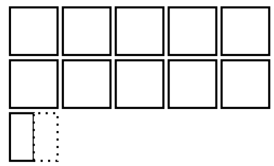
No calculators. Protractors, rulers, and compasses are permitted.

Answer format overview:

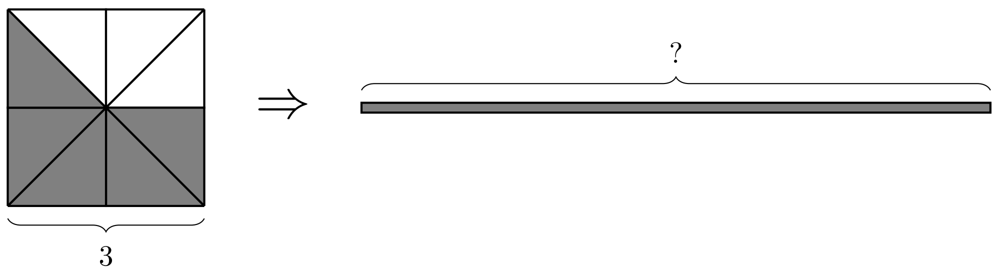
- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.

1. What is the value of $31^2 - 21^2 + 11^2 - 1^2$?

2. If Aedan bakes 25 identical brownie pieces, and Brian eats 10 and a half of those pieces, what percentage of the brownies did Brian eat? **If the answer is $x\%$, write only x as your answer.**



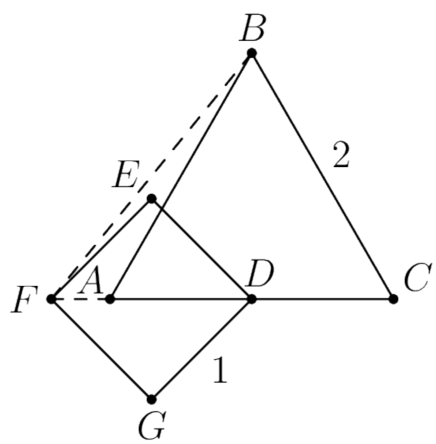
3. A square shaped pizza dough with side length 3 is divided into 8 slices of equal area. Three slices are removed and then the rest of the pizza dough is molded (preserving the area) into a long rectangular pizza dough with one side of length $\frac{1}{8}$. What is the length of the longer side?



4. Jeslyn writes down five numbers whose arithmetic average is 6. The first two numbers are 3 and 7, the third number is half the fifth number, and the fourth number is equal to the third number. What was the third number that Jeslyn wrote down?

5. How many positive even numbers less than 50 are there whose digits sum to an even number?

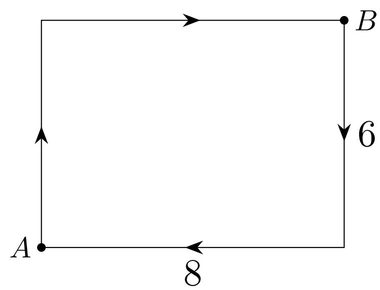
6. Let triangle $\triangle ABC$ be an equilateral triangle with side length 2, and let D be the midpoint of side $\overline{AC}$, as shown below. If $DEFG$ is a square with side length 1 such that A lies on diagonal $\overline{DF}$. What is the value of BF^2?



7. Helena writes down the number 0 on a chalkboard. Then, every minute afterwards, she counts how many digits in total are on the board and writes down that number. She repeats this until she has written 50 separate numbers on the board (including the first number, 0). For example, if $0, 3, 12$, and 147 were written on the board, there would be four numbers and seven digits. What is the last number Helena writes?

8. Isaac picks a number among 1, 2, 3, 4, 5 uniformly at random. Preston picks a number among 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 uniformly at random. What is the probability that Preston picks a strictly larger number than Isaac?

9. Jonathan and Ethan are racing around a rectangular track that is 6 units wide and 8 units long. Jonathan can finish a lap in 4 seconds, while Ethan can finish a lap in 7 seconds. They race one lap around the track, starting at point A and going clockwise. However, once Ethan reaches point B, he cheats by running off of the track, taking the most direct path back to A, at the same speed as before. He still loses the race to Jonathan, who does not cheat. How far away was Ethan from Jonathan when Jonathan finishes the race, *in units*?



10. Let x and y be positive integers such that $150x^2y$ and $60xy^2$ are both perfect squares and $300xy$ is a perfect cube. Compute the minimum possible value of $x + y$.

11. Aditya chooses a random permutation of the letters that make up "REPOSITORY". What is the probability that Aditya's permutation contains the word "OR" twice? For example, "ORSITYORPE" is one such permutation, but "OROEPSITRY" is not.

12. Call a positive integer n *basic* if there is a positive integer $b > 1$ such that n can be written with b digits in base b (with no leading 0s). For example, 3 is *basic* because it can be written with 2 digits in base 2 as 11_2. How many positive integers $n \leq 2025$ are *basic*?

13. Let $\varphi = \frac{1+\sqrt{5}}{2}$. There exist positive integers a and b such that

$$\sqrt{\varphi} + \sqrt{\frac{1}{\varphi}} = \sqrt{a + \sqrt{b}}.$$

Find $a + b$.

14. Distinct points A, B, T, and D lie on a line such that $AB = BT = TD = 40$. Points E and F satisfy $AE = DF = 48$ and $TE = BF = 64$, with line segments $\overline{TE}$ and $\overline{BF}$ intersecting at a point M. What is the area of triangle $\triangle BMT$?

15. For integers a and b, define the binary operation $\bigstar$ by

$$a \bigstar b = a + b + ab.$$

There exists an associative operation $\blacktriangledown$, meaning that $(a \blacktriangledown b) \blacktriangledown c = a \blacktriangledown (b \blacktriangledown c)$ for all real numbers a, b and c, such that whenever x is a non-negative integer,

$$x \bigstar y = \underbrace{y \blacktriangledown \cdots \blacktriangledown y \blacktriangledown y}_{x \ \blacktriangledown\text{'s}}.$$

Compute $20 \blacktriangledown 25$.

16. Square $ABCD$ has side length 2. A circle is drawn such that it passes through C and is tangent to sides $\overline{AB}$ and $\overline{AD}$. The area of the overlapping region covered by both the circle and square can be written as $(a+b\sqrt{c})\pi+d+e\sqrt{f}$, where a, b, c, d, e and f are all integers, and c and f are square-free (they are not divisible by any perfect square greater than 1). Find $a+b+c+d+e+f$.

17. Consider a bee (denoted by X) in a rectangular honeycomb as seen below:

	1	2	3	4	5	6	7
A				X			
B							
C							
D							

In one move, the bee may move to an adjacent square via an up, down, left, or right move, and it can no longer move once it reaches row D. The bee cannot move outside the honeycomb. It cannot revisit a square it has already been to, and it cannot move more than six times. Find the number of different paths the bee can take from its starting point to row D.

18. What is the least positive number of zeroes that can be concatenated to the end of 2025 such that the sum of the even factors of the resulting number is divisible by 45?
Here "concatenated" means writing zeroes at the end of the number.
For example: Concatenating one zero to '2025' gives '20250', concatenating two zeroes gives '202500', and so on.

19. What is the greatest possible value of C satisfying the property that the following system of equations

$$x^2 = y + C, \quad y^2 = x + C$$

has exactly 2 real solutions, and all solutions are real?

20. Let ω_1 be a circle with center O and radius 4 and ω_2 be a circle with center P and radius 1 such that ω_1 and ω_2 are externally tangent to each other and both tangent to line $\overleftrightarrow{AB}$, with ω_1 tangent at A, and ω_2 tangent at B. Point D lies on ω_2 such that O, P, and D are collinear and D is not on ω_1. Line ℓ is tangent to circle ω_2 at D. Let C be the point of intersection of line $\overleftrightarrow{OA}$ and line ℓ, and let K be the point of intersection of line $\overleftrightarrow{AB}$ and line ℓ. If Q is the center of the inscribed circle of triangle $\triangle AKC$, compute the area of $\triangle OPQ$.