Berkeley mini Math Tournament 2025

Relay Round

April 12, 2025

Time limit: 40 minutes.

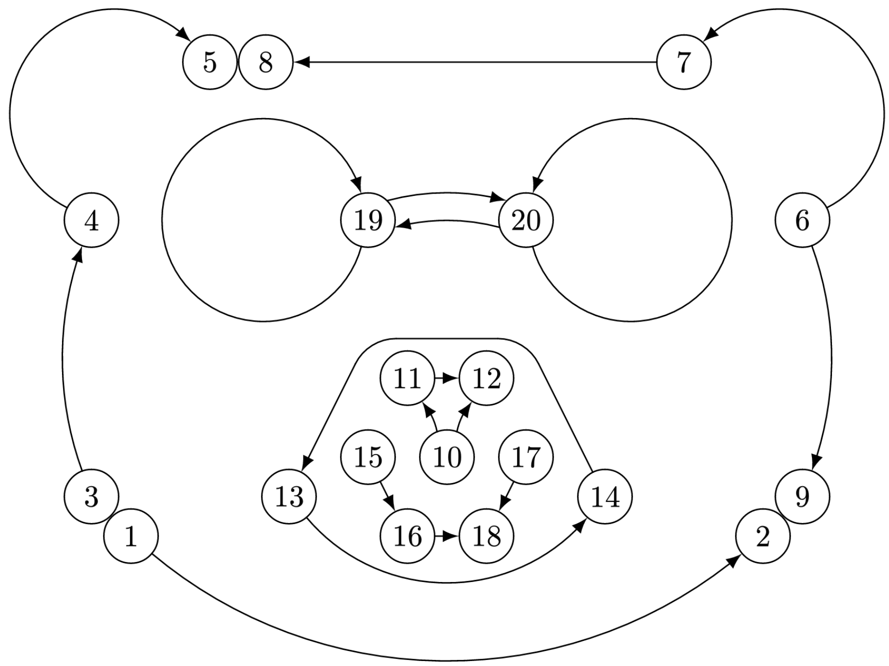
Instructions: For this test, you will work in teams of up to five to solve 7 sets of short answer questions, with 20 questions in total. You may work on any problem from any set at any time. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.

Note: Some questions may depend on other questions' answers. The drawing at the top of every set indicates which other answers are used for which problems. The integrated drawing of all 7 sets will be shown on the top of the first page. All answers for this round are numerical values (not in terms of any variables).

No calculators. Protractors, rulers, and compasses are permitted.

Answer format overview:

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.



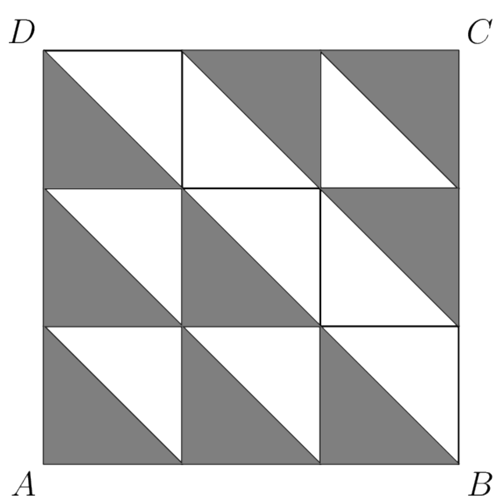
Set 1

$$1 \longrightarrow 2$$

1. What is the largest number Jessica can create using each of $1, 2$, and 6 at most once along with the operations $+$, $-$, $\times$, $\div$, and parentheses? Note that combining digits (e.g., making 16 from 1 and 6) is not allowed.

2. Let N_1 be the answer to Problem 1.

   If the area of square $ABCD$ below is N_1, find the area of the shaded region.



Set 2

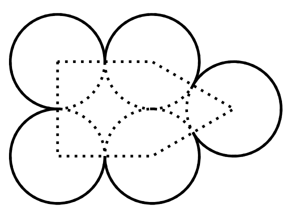
$$3 \longrightarrow 4 \longrightarrow 5$$

3. An ancient numbering system has three symbols: smiles [$\smile$], frowns [$\frown$], and dots [$\cdot$]. Each number is represented by a group of dots on top of any amount of frowns or smiles. Each dot counts as 1, each frown counts as 5, and each smile counts as 20. For example, $\overset{\cdot}{\frown}$ represents 6. What is the numerical value of the following expression?

$$\overset{\cdot\cdot}{\overset{\smile}{\smile}} \times (\overset{\cdot\cdot}{\frown} \times \overset{\cdot\cdot}{\frown} - \cdot) + (\overset{\cdot\cdot}{\smile} - \overset{\cdot\cdot}{\frown} - \overset{\cdot\cdot}{\frown} + \cdot)$$

4. Let N_3 be the **square root** of the answer to Problem 3.

   Consider a convex pentagon with two adjacent right angles and all sides having integer length N_3, as shown in the diagram below. Evan draws a circle centered at each vertex such that each circle is tangent to the circles centered at adjacent vertices and all circles are congruent. If the perimeter of the resulting shape, shown in bold, is $\frac{a\pi}{2}$, find a.



5. Let N_4 be the **rightmost digit** of the answer to Problem 4.

   How many positive factors of $\frac{2025}{N_4}$ have no digits that are prime numbers?

Set 3

$$\begin{tikzcd} & 7 \arrow[r] & 8 \\ 6 \arrow[ur] \arrow[dr] & & \\ & 9 & \end{tikzcd}$$

6. Aarush writes down the following equations involving three positive numbers A, B, and K:

$$K \cdot A \cdot B \cdot A \cdot B = \frac{4}{9},$$
$$A \cdot B \cdot A \cdot K \cdot A \cdot B \cdot A = \frac{1}{9},$$
$$B \cdot A \cdot A \cdot B \cdot A \cdot A = \frac{1}{36},$$

   where the left hand side of each equation is a product of A's, B's, and K's. Determine the value of Aarush's constant, defined as $B \cdot A \cdot B \cdot B \cdot 1 \cdot 3 \cdot 5$.

7. Let N_6 be the **denominator** when the answer to Problem 6 is written in simplest form. If your answer to Problem 6 is a mixed number, convert it to an improper fraction to find N_6.

   Oliver is planning a "world tour" in which he plans on visiting N_6 places, including his starting position, on the surface of the Earth, selected uniformly at random. Assuming Earth is a perfect sphere and that the equator divides it into two congruent hemispheres, what is the probability that Oliver must cross the equator at some point during his tour?

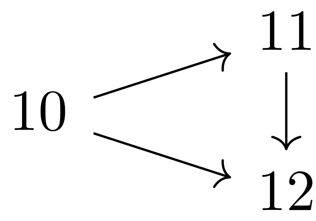
8. Let N_7 be the answer to Problem 7.

   How many circles are tangent to all three lines $y = \frac{x}{N_7}$, $y = N_7(x+1)$, and $y = N_7 - x$?

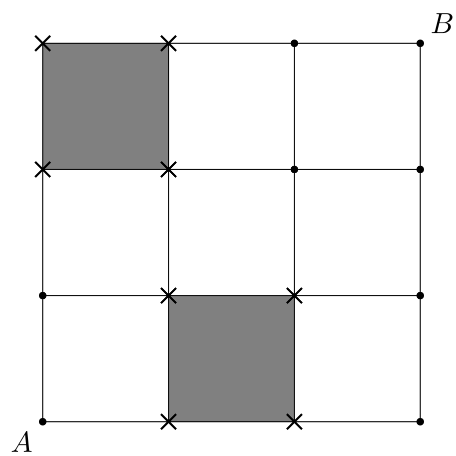
9. Let N_6 be the answer to Problem 6.

   Define the *naive sum*, denoted by $\oplus$, of any two simplified fractions to be $\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$, where a, b, c, and d must be positive integers. For example, $\frac{1}{3} \oplus \frac{3}{5} = \frac{4}{8} = \frac{1}{2}$. Let $\frac{p}{q}$ be the smallest simplified fraction such that $N_6 \oplus \frac{p}{q} \geq N_6$. Find $\frac{p}{q}$.

Set 4



10. Arthur the Ant lives at point A on a 4×4 grid of points and can move only to adjacent points connected by line segments. However, Shrey the Snail blocks two of the nine squares, making all vertices of those squares inaccessible to Arthur. If the order that Shrey blocked them does not matter, how many ways can Shrey choose two distinct squares to block such that Arthur can still travel from A to B?



One example way that leaves no possible path from A to B.

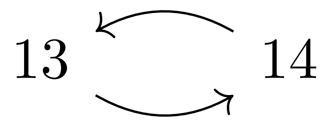
11. Let N_{10} be the answer to Problem 10.

    Rectangle $ABCD$ has $AB = 18$ and $BC = 24$. Harsh draws point X on $\overline{AD}$ such that $AX = N_{10}$. Let Y and Z be points on $\overline{BD}$ such that $BY = YZ = ZD$. Find the area of triangle $\triangle XYZ$.

12. Let N_{10} be the answer to Problem 10 and N_{11} be the answer to Problem 11.

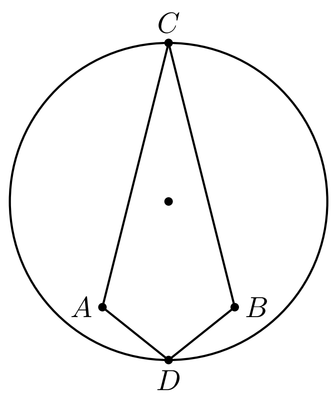
    Call a pentagon $ABCDE$ *tri-right* if all of its interior angles are less than $180°$, all of its side lengths are positive integers, $AB = CD$, and $\angle B = \angle C = \angle E = 90°$. Reflections and rotations of a pentagon are not considered distinct. What is the number of distinct *tri-right* pentagons with perimeter at most $4N_{10}$ such that side length AB is a multiple of $\sqrt{N_{11}}$?

Set 5



13. Let N_{14} be the answer to Problem 14.

    Let O be a circle with radius N_{14} as shown below, and let $\overline{CD}$ be a diameter of circle O. Let A and B be points inside the circle such that $\overline{AB} \perp \overline{CD}$. If the ratio of the area of circle O to the area of quadrilateral $ACBD$ is $\frac{6}{5}\pi$, what is AB?



14. Let N_{13} be the answer to Problem 13.

    Mary, Sabine, Meghan, and Justin each take a quiz. The sum of Mary's score, Sabine's score, and Meghan's score is 140. The average of all four scores is N_{13}. What score did Justin get?

Set 6

$$
\begin{tikzcd}
15 \arrow[r] & 16 \arrow[dr] & \\
& & 18 \\
& 17 \arrow[ur] &
\end{tikzcd}
$$

15. A team of 60 gardeners is planting trees. The 60 gardeners are split into groups A, B, and C with a, b, and c members, respectively. If, every day, each member in group A plants a trees, each in B plants b trees, and each in C plants c trees, then they can plant 10320 trees in 5 days. If their planting rates are swapped so that each person in group A plants b trees, each in B plants c, and each in C plants a, how many trees can the 60 gardeners plant in one day?

16. Let N_{15} be the **largest prime factor** of the answer to Problem 15.

    Viraj lists out all the fractions $\frac{a}{b}$ such that the sum of positive integers a and b is less than or equal to N_{15}^2. He then erases all the expressions that are not in simplest form. Note that for this problem, $\frac{5}{1}$ and $\frac{2}{4}$ are **not** in simplest form, but $\frac{7}{3}$ is in simplest form. If the product of all the remaining fractions is $\frac{1}{P}$, determine the largest perfect cube that is a factor of P.

17. The number 2025 can be expressed as a sum using only prime numbers between 100 and 110. What is the largest number of times that 109 can appear in such a sum? Note that the sum does **not** need to use each prime at least once.

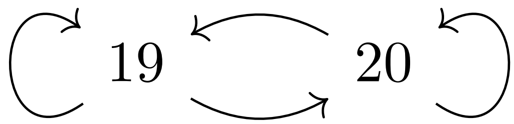
18. Let N_{16} be the answer to Problem 16 and N_{17} be the answer to Problem 17.

    Given a right triangle with side lengths a, b, and c, we can transform it into a new right triangle using the following two-step transformation:

    1. Remove one of the sides of the triangle.
    2. Create a right triangle with the remaining two sides as the legs.

    If we start with a right triangle with side lengths $1, 1, \sqrt{2}$, determine the minimum number of transformations needed to obtain a right triangle with legs of length $\sqrt[3]{N_{16}}$ and N_{17}.

Set 7



19. Let N_{19} be the answer to this problem and N_{20} be the answer to Problem 20.

    Square $ABCD$ has side length N_{20} and has points E and H inside such that $\overline{EH}$ is parallel to $\overline{AB}$. Point F is inside $ABCD$ such that the ray $\overrightarrow{EF}$ intersects $\overline{AB}$ at point F'. Point G is on $\overline{BC}$ such that $BG = \sqrt{N_{19}}$. Given that $\angle GFF' = 90°$ and $\angle FF'A = \angle FF'C = 45°$, determine the area of heptagon $AEFGCHD$.

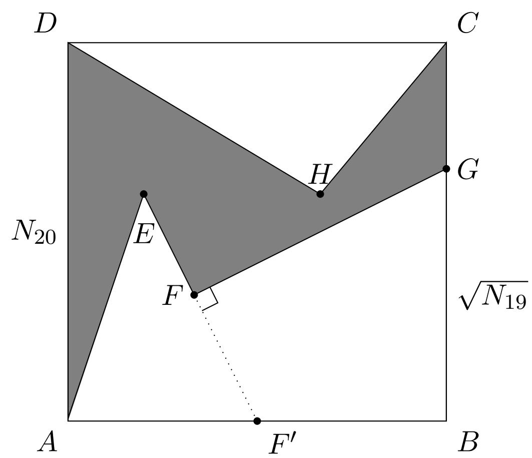


Diagram not drawn to scale.

20. Let N_{19} be the answer to Problem 19 and N_{20} be the answer to this problem.

    Two lines with equations $y = N_{20}x + a$ and $y = N_{19}x + b$ intersect at a point with x-coordinate equal to 1 in the coordinate plane. What is the value of N_{20} for which $b - a$ is maximized?