# Berkeley mini Math Tournament 2025

## Individual Round

April 12, 2025

**Time limit:** 60 minutes.

**Instructions:** This test contains 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No calculators.** Protractors, rulers, and compasses are permitted.

**Answer format overview:**

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as $7^8$.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as $\pi$ in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.

1. Timothe is reserving 100 classrooms for BmMT this year, and each room has exactly one purpose: testing, grading, or activities. Half of the rooms are for testing and five rooms are for grading. How many rooms are left for activities?

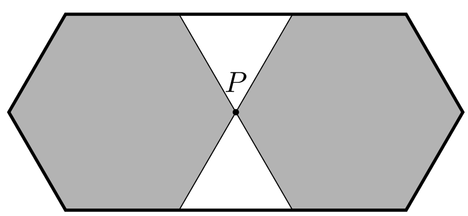
2. Points $A$, $B$, and $C$ lie on a straight line, not necessarily in that order. The distance between $A$ and $B$ is 5, and the distance between $A$ and $C$ is 3. What is the sum of all distinct possible distances between $B$ and $C$?

3. Benji writes down the first five odd positive integers and erases one of them. Kiran writes down the first five even positive integers and erases one of them. Benji notices that the sum of his four remaining numbers is equal to the sum of Kiran's four remaining numbers. If the number that Benji erased is $B$ and the number that Kiran erased is $K$, find $K - B$.

4. A bag contains two red marbles, two blue marbles, and two green marbles. Richard removes marbles from the bag one at a time without replacement. What is the least number of marbles Richard must remove to guarantee that two of the marbles that he has taken out are the same color?

5. Four consecutive odd integers sum to $-16$. What is the product of these four integers?

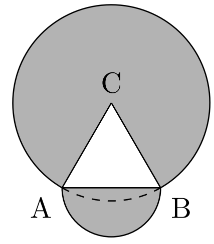
6. Point $P$ is a vertex shared by two congruent regular hexagons (both shaded) and two equilateral triangles (both unshaded) with the same side length, as shown below. Together, these shapes form a larger, non-regular hexagon (drawn with a thick border). What is the ratio of the **combined** area of both regular hexagons (the shaded area) to the area of the large hexagon? The answer may be expressed in any form.




7. How many positive factors of 120 are divisible by 12? Note that 1 and 120 are positive factors of 120.

8. Luke the frog lives on a pond with 5 lilypads, labeled 1 through 5. He starts at lilypad 1 and, at every step, hops to a lilypad with a larger number, chosen uniformly at random. Luke continues hopping until he reaches lilypad 5. What is the probability that it takes Luke exactly 2 steps to reach lilypad 5?

9. Around equilateral triangle $\triangle ABC$ shown below, a circle centered at $C$ with radius $\overline{CA}$ and a semicircle with diameter $\overline{AB}$ are drawn. If the total area of the shaded region (the whole shape excluding $\triangle ABC$) is $138\pi$, what is the perimeter of $\triangle ABC$?



10. Aaron has 1 penny, 1 nickel, 1 dime, and 1 quarter. These coins are worth 1, 5, 10, and 25 cents, respectively. Aaron makes a list of all the different numbers of cents he can make with some or all of his coins. If Aaron adds up all the numbers in his list, what is the result **in cents**?

11. Find the sum of all numbers $n$ such that the equation $x^2 - nx + 80 = 0$ has two positive integer solutions, and one solution is an integer multiple of the other.

12. A positive integer $n$ is *two-cool* if the decimal expansion of $n/250$ has exactly two digits past the decimal point, excluding trailing zeros. For example, $10/250 = 0.04$ has exactly two digits past the decimal point, and $100/250 = 0.4$ has exactly one digit past the decimal point, so 10 is *two-cool* but 100 is not. How many positive integers less than 250 are *two-cool*?

13. Points $A$, $B$, and $C$ lie on a circle. Let segment $\overline{BC}$ extend through $C$ to point $D$ such that $\overline{AD}$ is tangent to the circle. If $AC = 4$, $BC = 9$, and $\angle ACD = 90^\circ$, what is $CD$?

14. Define a sequence of positive integers $a_1, a_2, a_3, \dots$ such that $a_1 = 1$ and

$$a_i = 81a_{i-1} + i$$

for all integers $i \geq 2$. What is the smallest positive integer $n$ such that $a_n$ is divisible by 20?

15. Nikki chooses three distinct square cells, $A$, $B$, and $C$, from a $3 \times 3$ square grid uniformly at random. What is the probability that square $C$ is contained within the rectangle whose opposite corners are squares $A$ and $B$? The answer may be expressed in any form. Examples of rectangles with opposite corners at $A$ and $B$ are shown as shaded regions below.


16. Let $a$, $b$, and $c$ be positive integers such that $\{a, b, c, 2025^2\}$ is a geometric sequence that is strictly increasing, meaning $a < b < c < 2025^2$. Find the number of distinct possible values of $a$.

17. Let $(a, b, c, d, e)$ be a permutation of $(2, 3, 4, 5, 6)$. For example, a possible permutation is $(a, b, c, d, e) = (5, 3, 6, 2, 4)$. What is the maximum possible value of $a - ab + abc - abcd + abcde$, where $ab$, $abc$, $abcd$, and $abcde$ all represent multiplication (e.g., $ab = a \cdot b$), not permutations?

18. Let $ABCDEF$ be a regular hexagon. Let $P$ be a point on segment $\overline{BF}$, and let line $\overleftrightarrow{CP}$ intersect segment $\overline{AF}$ at $Q$. If $AB = 12$ and $\frac{BP}{BF} = \frac{3}{4}$, what is $PQ$?

19. Danielle calculates the ones digit of each of the 2025 integers $1^{1!}, 2^{2!}, 3^{3!}, \cdots, 2025^{2025!}$. If Danielle adds up all these ones digits, what is the resulting value?

20. There exist strictly increasing arithmetic sequences of real numbers, $\{a, b, c\}$ and $\{p, q, r\}$, having the properties that $q$ is a positive integer greater than 1 and that the equation $x^3 - ax^2 + bx - c = 0$ has solutions $p, q$, and $r$. Over all such pairs of increasing arithmetic sequences, what is the least possible value of $p + q + r$?