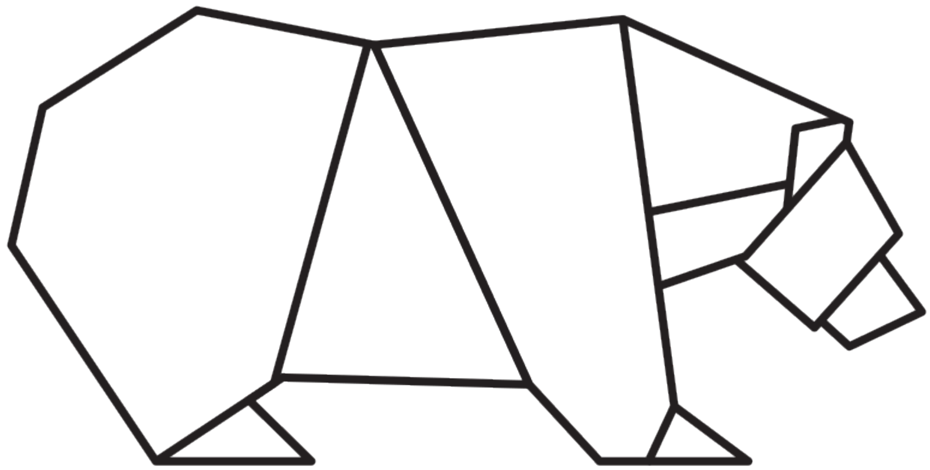
Berkeley Math Tournament 2024

Geometry Test



November 2, 2024

Time limit: 60 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators. Protractors, rulers, and compasses are permitted.

Answer format overview:

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π, e, $\sin(10°)$, and $\ln 2$ in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.

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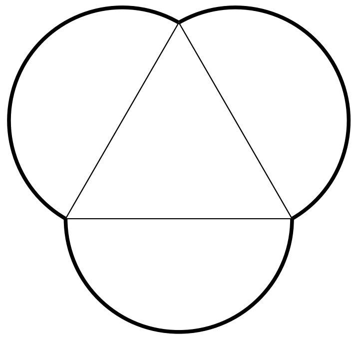
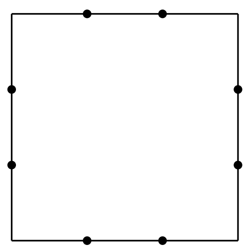
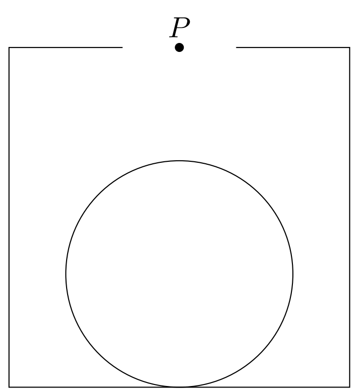
1. Andrew has three identical semicircular mooncake halves, each with radius 3, and uses them to construct the following shape, which contains an equilateral triangle in the center. Compute the perimeter around this shape, in bold below.



2. On a chalkboard, Benji draws a square with side length 6. He then splits each side into 3 equal segments using 2 points for a total of 12 segments and 8 points. After trying some shapes, Benji finds that by using a circle, he can connect all 8 points together. What is the area of this circle?



3. A square with side length 6 has a circle with radius 2 inside of it, with the centers of the square and circle vertically aligned. Aarush is standing 4 units directly above the center of the circle, at point P. What is the area of the region inside the square that he can see?



4. Two circles, ω_1 and ω_2, are internally tangent at A. Let B be the point on ω_2 opposite of A. The radius of ω_1 is 4 times the radius of ω_2. Point P is chosen on the circumference of ω_1 such that the ratio $\frac{AP}{BP} = \frac{2\sqrt{3}}{\sqrt{7}}$. Let O denote the center of ω_2. Determine the ratio $\frac{OP}{AO}$.

5. Let U and C be two circles, and kite $BERK$ have vertices that lie on U and sides that are tangent to C. Given that the diagonals of the kite measure 5 and 6, find the ratio of the area of U to the area of C.

6. Let triangle $\triangle ABC$ be acute. Point D is the foot of the altitude of $\triangle ABC$ from A to $\overline{BC}$, and E is the foot of the altitude of $\triangle ABC$ from B to $\overline{AC}$. Let F denote the point of intersection between $\overline{BE}$ and $\overline{AD}$, and let G denote the point of intersection between $\overline{CF}$ and $\overline{DE}$. The areas of triangles $\triangle EFG$, $\triangle CDG$, and $\triangle CEG$ are 1, 4, and 3 respectively. Find the area of $\triangle ABC$.

7. In parallelogram $ABCD$, E is a point on $\overline{AD}$ such that $\overline{CE} \perp \overline{AD}$, F is a point on $\overline{CD}$ such that $\overline{AF} \perp \overline{CD}$, and G is a point on $\overline{BC}$ such that $\overline{AG} \perp \overline{BC}$. Let H be a point on $\overline{GF}$ such that $\overline{AH} \perp \overline{GF}$, and let J be the intersection of $\overleftrightarrow{EF}$ and $\overleftrightarrow{BC}$. Given that $AH = 8$, $AE = 6$, and $EF = 4$, compute CJ.

8. Points A, B, C, D, E, and F lie on a sphere with radius $\sqrt{10}$ such that lines $\overleftrightarrow{AD}$, $\overleftrightarrow{BE}$, and $\overleftrightarrow{CF}$ are concurrent at point P inside the sphere and are pairwise perpendicular. If $PA = \sqrt{6}$, $PB = \sqrt{10}$, and $PC = \sqrt{15}$, what is the volume of tetrahedron $DEFP$?

9. Let $\triangle ABC$ be a triangle with incenter I, and let M be the midpoint of $\overline{BC}$. Line $\overleftrightarrow{AM}$ intersects the circumcircle of triangle $\triangle IBC$ at points P and Q. Suppose that $AP = 13, AQ = 83$, and $BC = 56$. Find the perimeter of $\triangle ABC$.

10. The incircle of scalene triangle $\triangle ABC$ is tangent to $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ at points D, E, and F, respectively. The line $\overleftrightarrow{EF}$ intersects line $\overleftrightarrow{BC}$ at P and line $\overleftrightarrow{AD}$ at Q. The circumcircle of $\triangle AEF$ intersects line $\overleftrightarrow{AP}$ again at point $R \neq A$. If $QE = 3$, $QF = 4$, and $QR = 8$, find the area of triangle $\triangle AEF$.