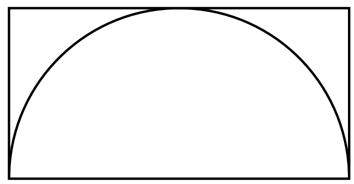
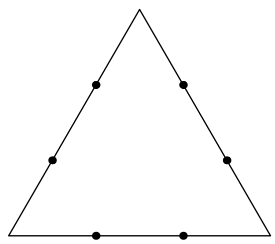
Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

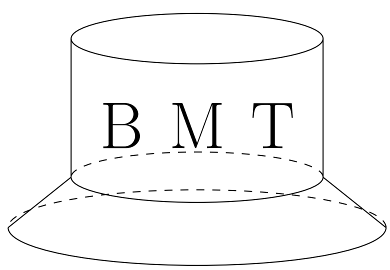
1. A semicircle of radius 2 is inscribed inside of a rectangle, as shown in the diagram below. The diameter of the semicircle coincides with the bottom side of the rectangle, and the semicircle is tangent to the rectangle at all points of intersection. Compute the length of the diagonal of the rectangle.



2. Consider an equilateral triangle with side length 9. Each side is divided into 3 equal segments by 2 points, for a total of 6 points. Compute the area of the circle passing through these 6 points.



3. Jingyuan is designing a bucket hat for BMT merchandise. The hat has the shape of a cylinder on top of a truncated cone, as shown in the diagram below. The cylinder has radius 9 and height 12. The truncated cone has base radius 15 and height 4, and its top radius is the same as the cylinder's radius. Compute the total volume of this bucket hat.



4. Let ω be a circle with center O and radius 8, and let A be a point such that $AO = 17$. Let P and Q be points on ω such that line segments $\overline{AP}$ and $\overline{AQ}$ are tangent to ω. Let B and C be points chosen on $\overline{AP}$ and $\overline{AQ}$, respectively, such that $\overline{BC}$ is also tangent to ω. Compute the perimeter of triangle $\triangle ABC$.

5. Triangle $\triangle ABC$ has side lengths $AB = 8$, $BC = 15$, and $CA = 17$. Circles ω_1 and ω_2 are externally tangent to each other and within $\triangle ABC$. The radius of circle ω_2 is four times the radius of circle ω_1. Circle ω_1 is tangent to $\overline{AB}$ and $\overline{BC}$, and circle ω_2 is tangent to $\overline{BC}$ and $\overline{CA}$. Compute the radius of circle ω_1.

6. In triangle $\triangle ABC$, let M be the midpoint of $\overline{AC}$. Extend $\overline{BM}$ such that it intersects the circumcircle of $\triangle ABC$ at a point X not equal to B. Let O be the center of the circumcircle of $\triangle ABC$. Given that $BM = 4MX$ and $\angle ABC = 45^\circ$, compute $\sin(\angle BOX)$.

7. A tetrahedron has three edges of length 2 and three edges of length 4, and one of its faces is an equilateral triangle. Compute the radius of the sphere that is tangent to every edge of this tetrahedron.

8. A circle intersects equilateral triangle $\triangle XYZ$ at A, B, C, D, E, and F such that points X, A, B, Y, C, D, Z, E, and F lie on the equilateral triangle in that order. If $AC^2 + CE^2 + EA^2 = 1900$ and $BD^2 + DF^2 + FB^2 = 2092$, compute the positive difference between the areas of triangles $\triangle ACE$ and $\triangle BDF$.

9. Let triangle $\triangle ABC$ be acute, and let point M be the midpoint of $\overline{BC}$. Let E be on line segment $\overline{AB}$ such that $\overline{AE} \perp \overline{EC}$. Then, suppose T is a point on the other side of $\overleftrightarrow{BC}$ as A is such that $\angle BTM = \angle ABC$ and $\angle TCA = \angle BMT$. If $AT = 14$, $AM = 9$, and $\frac{AE}{AC} = \frac{2}{7}$, compute BC.

10. Let triangle $\triangle ABC$ have circumcenter O and circumradius r, and let ω be the circumcircle of triangle $\triangle BOC$. Let F be the intersection of $\overleftrightarrow{AO}$ and ω not equal to O. Let E be on line $\overleftrightarrow{AB}$ such that $\overline{EF} \perp \overline{AE}$, and let G be on line $\overleftrightarrow{AC}$ such that $\overline{GF} \perp \overline{AG}$. If $AC = \frac{65}{63}$, $BC = \frac{24}{13}r$, and $AB = \frac{126}{65}r$, compute $AF \cdot EG$.