1. Wen finds 17 consecutive positive integers that sum to 2023. Compute the smallest of these integers.

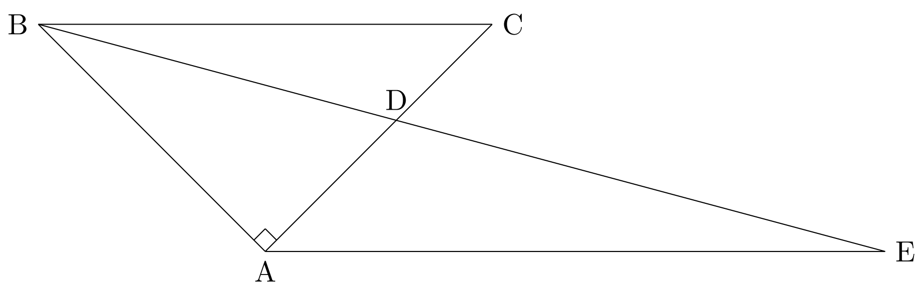
   Answer: 111

   Solution: Notice that the average of these integers must be $\frac{2023}{17} = 119$. Since they are consecutive, we know that the middle, 9th, number out of our 17 integers must be 119. This will be 8 more than the smallest integer, so our answer is $119 - 8 = \boxed{111}$.

2. Triangle $\triangle ABC$ has $\angle ABC = \angle BCA = 45^\circ$ and $AB = 1$. Let D be on $\overline{AC}$ such that $\angle ABD = 30^\circ$. Let $\overleftrightarrow{BD}$ and the line through A parallel to $\overleftrightarrow{BC}$ intersect at E. Compute the area of $\triangle ADE$.

   Answer: $\mathbf{\frac{3+\sqrt{3}}{12}}$

   Solution:



   Triangles $\triangle ADE$ and $\triangle CDB$ are similar. Thus, the area of $\triangle ADE$ is $(\frac{AD}{DC})^2$ times the area of $\triangle CDB$. Since $\angle ABD = 30^\circ$ and $AB = 1$, we have $AD = \frac{1}{\sqrt{3}}$ and $DC = 1 - \frac{1}{\sqrt{3}}$. Thus, the area of $\triangle ADE$ is $\frac{1}{2}\left(1 - \frac{1}{\sqrt{3}}\right)\left(\frac{1/\sqrt{3}}{1-(1/\sqrt{3})}\right)^2 = \boxed{\frac{3+\sqrt{3}}{12}}$.

3. Mataio has a weighted die numbered 1 to 6, where the probability of rolling a side n for $1 \leq n \leq 6$ is inversely proportional to the value of n. If Mataio rolls the die twice, what is the probability that the sum of the two rolls is 7?

   Answer: $\mathbf{\frac{40}{343}}$

   Solution: The probability of rolling a given side n is $\frac{k}{n}$ for some constant k. So the probability that rolling the die twice yields a sum of 7 is $2 \cdot (\frac{k}{1} \cdot \frac{k}{6} + \frac{k}{2} \cdot \frac{k}{5} + \frac{k}{3} \cdot \frac{k}{4}) = \frac{7k^2}{10}$
   We can solve for k using the following equation
   $1 = \frac{k}{1} + \frac{k}{2} + \frac{k}{3} + \frac{k}{4} + \frac{k}{5} + \frac{k}{6}$
   Solving, we get $k = \frac{20}{49}$, and plugging into $\frac{7k^2}{10}$ gives the final answer of $\boxed{\frac{40}{343}}$

4. Let $N = 2^{18} \cdot 3^{19} \cdot 5^{20} \cdot 7^{21} \cdot 11^{22}$. Compute the number of positive integer divisors of N whose units digit is 7.

   Answer: 2530

   Solution: Any divisor n can be written as

$$n = 2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot 11^e,$$

where a, b, c, d, e are nonnegative integers bounded above. To count divisors n with units digit 7, we immediately require $a = c = 0$. Now, the key trick is that 3 generates $(\mathbb{Z}/10\mathbb{Z})^\times$ because

$$\{3^0, 3^1, 3^2, 3^3\} \equiv \{1, 3, 9, 7\} \pmod{10},$$

so for any choice of d and e, there is a unique value of $b \pmod 4$ which give $n = 3^b \cdot 7^d \cdot 11^e$ with units digit 7. Because $0 \le b < 19$, this amounts to five different choices for b for any choices of (d, e).

To finish, we note that d has 22 options, and e has 23 options, so our answer is $5 \cdot 22 \cdot 23 = 110 \cdot 23 = \boxed{2530}$.

5. Compute the real solution for x to the equation $(4^x + 8)^4 - (8^x - 4)^4 = (4 + 8^x + 4^x)^4$.

Answer: $\mathbf{\frac{2}{3}}$

Solution: Let $a = 4^x + 8$ and $b = 8^x - 4$. Then the equation becomes $a^4 - b^4 = (a + b)^4$. Then $(a^2 - b^2)(a^2 + b^2) = (a + b)^4$, and so $(a - b)(a + b)(a^2 + b^2) = (a + b)^4$. Note that $a + b = (4^x + 8) + (8^x - 4) = 4^x + 8^x + 4 > 0$. Thus, we can divide by $a + b$ on both sides to get that $(a - b)(a^2 + b^2) = (a + b)^3$. Expanding the LHS and RHS gives $a^3 - b^3 - a^2b + ab^2 = a^3 + 3a^2b + 3ab^2 + b^3$, which after simplification yields $2b^3 + 4a^2b + 2ab^2 = 0$. We can factor out a b and we have $b(2b^2 + 4a^2 + 2ab) = 0$. This gives a solution of $b = 0$, which implies that the only real solution here is $a = 0, b = 0$. The expression $2b^2 + 4a^2 + 2ab$ has no other real solutions. We can show this by factoring it as $2(2a^2 + ab + b^2)$: applying the quadratic formula by treating the b terms as constants would imply the discriminant, $-7b^2$, is negative. Since $b = 0$ is our only possibility, we have $8^x - 4 = 0$, so $8^x = 4$, which means $x = \boxed{\frac{2}{3}}$ is the solution for x.