Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Lakshay chooses two numbers, m and n, and draws two lines, $y = mx + 3$ and $y = nx + 23$. Given that the two lines intersect at $(20, 23)$, compute $m + n$.

2. For real numbers x and y, suppose that $|x| - |y| = 20$ and $|x| + |y| = 23$. Compute the sum of all possible distinct values of $|x - y|$.

3. Consider two geometric sequences $16, a_1, a_2, \ldots$ and $56, b_1, b_2, \ldots$ with the same common nonzero ratio. Given that $a_{2023} = b_{2020}$, compute $b_6 - a_6$.

4. Let $f(x)$ be a continuous function over the real numbers such that for every integer n, $f(n) = n^2$ and $f(x)$ is linear over the interval $[n, n + 1]$. There exists a unique two-variable polynomial g such that $g(x, \lfloor x \rfloor) = f(x)$ for all x. Compute $g(20, 23)$. (Here, $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x. For example, $\lfloor 2 \rfloor = 2$ and $\lfloor -3.5 \rfloor = -4$.)

5. Let p, q, and r be the three roots of the polynomial $x^3 - 2x^2 + 3x - 2023$. Suppose that the polynomial $x^3 + Bx^2 + Mx + T$ has roots $p + q$, $p + r$, and $q + r$ for real numbers B, M, and T. Compute $B - M + T$.

6. Define a sequence $a_0, a_1, a_2, \ldots$ recursively by $a_0 = 0$, $a_1 = 1$, and $a_{n+2} = a_{n+1} + xa_n$ for each $n \geq 0$ and some real number x. The infinite series
$$\sum_{n=0}^{\infty} \frac{a_n}{10^n} = 1.$$
Compute x.

7. Nikhil constructs a list of all polynomial pairs $(a(x), b(x))$ with real coefficients such that $a(x)$ has higher degree than $b(x)$ and $a(x)^2 + b(x)^2 = x^{10} + 1$. Danielle takes Nikhil's list and adds all polynomial pairs that satisfy the same conditions but have complex coefficients. If Nikhil's original list had N pairs and Danielle added D pairs, compute $D - N$.

8. Compute the smallest real t such that there exist constants a, b for which the roots of $x^3 - ax^2 + bx - \frac{ab}{t}$ are the side lengths of a right triangle.

9. A sequence of real numbers $\{x_n\}$ satisfies the recursion $x_{n+1} = 4x_n - 4x_n^2$, where $n \geq 1$. If $x_{2023} = 0$, compute the number of distinct possible values for x_1.

10. There exists a unique triple of integers (B, M, T) such that $B > T > M$ and
$$3B^2(3T - M) + 8M^2(B - T) + 3T^2(5M - B) - (2B^3 + 3M^3 + 4T^3) + 15BMT = 2023.$$
Compute $B + M + T$.