

Time limit: 15 minutes.

Instructions: This tiebreaker contains 5 short answer questions. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

No calculators.

1. The arithmetic mean of 2, 6, 8, and x is 7. The arithmetic mean of 2, 6, 8, x , and y is 9. What is the value of $y - x$?
2. Compute the radius of the largest circle that fits entirely within a unit cube.
3. Dexter and Raquel are playing a game with N stones. Dexter goes first and takes one stone from the pile. After that, the players alternate turns and can take anywhere from 1 to $x + 1$ stones from the pile, where x is the number of stones the other player took on the turn immediately prior. The winner is the one to take the last stone from the pile.
Assuming Dexter and Raquel play optimally, compute the number of positive integers $N \leq 2021$ where Dexter wins this game.
4. Let z_1, z_2 , and z_3 be the complex roots of the equation $(2z - 3\bar{z})^3 = 54i + 54$. Compute the area of the triangle formed by z_1, z_2 , and z_3 when plotted in the complex plane.
5. Let r, s, t, u be the distinct roots of the polynomial $x^4 + 2x^3 + 3x^2 + 3x + 5$. For $n \geq 1$, define $s_n = r^n + s^n + t^n + u^n$ and $t_n = s_1 + s_2 + \cdots + s_n$. Compute $t_4 + 2t_3 + 3t_2 + 3t_1 + 5$.