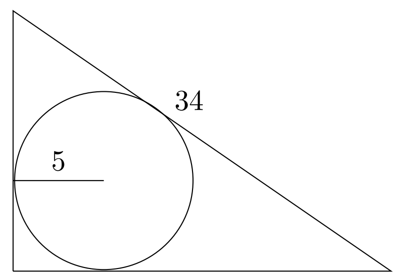
Time limit: 40 minutes.
Instructions: This test contains 15 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. A circle with radius 5 is inscribed in a right triangle with hypotenuse 34 as shown below. What is the area of the triangle? Note that the diagram is not to scale.



2. For how many values of x does $20^x * 18^x = 2018^x$?

3. 2018 people (call them A, B, C, ...) stand in a line with each permutation equally likely. Given that A stands before B, what is the probability that C stands after B?

4. Consider a standard (8-by-8) chessboard. Bishops are only allowed to attack pieces that are along the same diagonal as them (but cannot attack along a row or column). If a piece can attack another piece, we say that the pieces threaten each other. How many bishops can you place a chessboard without any of them threatening each other?

5. How many integers can be expressed in the form:

$$\pm 1 \pm 2 \pm 3 \pm 4 \cdots \pm 2018?$$

6. A rectangular prism with dimensions 20 cm by 1 cm by 7 cm with made with blue 1 cm unit cubes. The outside of the rectangular prism is coated in gold paint. If a cube is chosen at random and rolled, what is the probability that the size facing up is painted gold?

7. Suppose there are 2017 spies, each with $\frac{1}{2017}$th of a secret code. They communicate by telephone; when two of them talk, they share all information they know with each other. What is the minimum number of telephone calls that are needed for all 2017 people to know all parts of the code?

8. Alice is playing a game with 2018 boxes, numbered $1-2018$, and a number of balls. At the beginning, boxes $1-2017$ have one ball each, and box 2018 has $2018n$ balls. Every turn, Alice chooses i and j with $i > j$, and moves exactly i balls from box i to box j. Alice wins if all balls end up in box 1. What is the minimum value of n so that Alice can win this game?

9. Circles A, B, and C are externally tangent circles. Line PQ is drawn such that PQ is tangent to A at P, tangent to B at Q, and does not intersect with C. Circle D is drawn such that it passes through the centers of A, B, and C. Let R be the point on D furthest from PQ. If

$A, B,$and C have radius $3, 2,$and 1, respectively, the area of triangle PQR can be expressed in the form of $a+b\sqrt{c}$, where $a, b,$and c are integers with c not divisible by any prime square. What is $a+b+c$?

10. A rectangular prism has three distinct faces of area 24, 30, and 32. The diagonals of each distinct face of the prism form sides of a triangle. What is the triangle's area?

11. Ankit, Box, and Clark are playing a game. First, Clark comes up with a prime number less than 100. Then he writes each digit of the prime number on a piece of paper (writing 0 for the tens digit if he chose a single-digit prime), and gives one each to Ankit and Box, without telling them which digit is the tens digit, and which digit is the ones digit. The following exchange occurs:

    - Clark: There is only one prime number that can be made using those two digits.
    - Ankit: I don't know whether I'm the tens digit or the ones digit.
    - Box: I don't know whether I'm the tens digit or the ones digit.
    - Box: You don't know whether you're the tens digit or the ones digit.
    - Ankit: I don't know whether you're the tens digit or the ones digit.

    What was Clark's number?

12. Let $f : [0, 1] \to \mathbb{R}$ be a monotonically increasing function such that

$$f\left(\frac{x}{3}\right) = \frac{f(x)}{2}$$

$$f(1-x) = 2018 - f(x)$$

If $f(1) = 2018$, find $f\left(\frac{12}{13}\right)$.

13. Find the value of

$$\frac{1}{\sqrt{2}^1} + \frac{4}{\sqrt{2}^2} + \frac{9}{\sqrt{2}^3} \cdots$$

14. Let $F_1 = 0, F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$. Compute

$$\sum_{n=1}^{\infty} \frac{\sum_{i=1}^{n} F_i}{3^n}$$

15. Let triangle ABC have side lengths $AB = 13$, $BC = 14$, $AC = 15$. Let I be the incenter of ABC. The circle centered at A of radius AI intersects the circumcircle of ABC at H and J. Let L be a point that lies on both the incircle of ABC and line HJ. If the minimal possible value of AL is $\sqrt{n}$, where $n \in \mathbb{Z}$, find n.