1. Define a_n such that $a_1 = \sqrt{3}$ and for all integers i, $a_{i+1} = a_i^2 - 2$. What is a_{2016}?

2. Jennifer wants to do origami, and she has a square of side length 1. However, she would prefer to use a regular octagon for her origami, so she decides to cut the four corners of the square to get a regular octagon. Once she does so, what will be the side length of the octagon Jennifer obtains?

3. A little boy takes a 12 in long strip of paper and makes a Mobius strip out of it by tapping the ends together after adding a half twist. He then takes a 1 inch long train model and runs it along the center of the strip at a speed of 12 inches per minute. How long does it take the train model to make two full complete loops around the Mobius strip? A complete loop is one that results in the train returning to its starting point.

4. How many graphs are there on 6 vertices with degrees 1,1,2,3,4,5?

5. Let ABC be a right triangle with $AB = BC = 2$. Let ACD be a right triangle with angle $DAC = 30$ degrees and angle $DCA = 60$ degrees. Given that ABC and ACD do not overlap, what is the area of triangle BCD?

6. How many integers less than 400 have exactly 3 factors that are perfect squares?

7. Suppose $f(x, y)$ is a function that takes in two integers and outputs a real number, such that it satisfies
$$f(x, y) = \frac{f(x, y+1) + f(x, y-1)}{2}$$
$$f(x, y) = \frac{f(x+1, y) + f(x-1, y)}{2}$$
What is the minimum number of pairs (x, y) we need to evaluate to be able to uniquely determine f?

8. How many ways are there to divide 10 candies between 3 Berkeley students and 4 Stanford students, if each Berkeley student must get at least one candy? All students are distinguishable from each other; all candies are indistinguishable.

9. How many subsets (including the empty-set) of $\{1, 2..., 6\}$ do not have three consecutive integers?

10. What is the smallest possible perimeter of a triangle with integer coordinate vertices, area $\frac{1}{2}$, and no side parallel to an axis?

11. Circles C_1 and C_2 intersect at points X and Y. Point A is a point on C_1 such that the tangent line with respect to C_1 passing through A intersects C_2 at B and C, with A closer to B than C, such that $2016 \cdot AB = BC$. Line XY intersects line AC at D. If circles C_1 and C_2 have radii of 20 and 16, respectively, find the ratio of $\sqrt{1 + BC/BD}$.

12. Consider a solid hemisphere of radius 1. Find the distance from its center of mass to the base.

13. Consider an urn containing 51 white and 50 black balls. Every turn, we randomly pick a ball, record the color of the ball, and then we put the ball back into the urn. We stop picking when we have recorded n black balls, where n is an integer randomly chosen from $\{1, 2, \ldots, 100\}$ What is the expected number of turns?

14. Consider the set of axis-aligned boxes in $\mathbb{R}^d$, $B(a, b) = \{x \in \mathbb{R}^d : \forall i, a_i \leq x_i \leq b_i\}$ where $a, b \in \mathbb{R}^d$. In terms of d, what is the maximum number n, such that there exists a set of n points $S = \{x_1, ..., x_n\}$ such that no matter how one partition $S = P \cup Q$ with P, Q disjoint and P, Q can possibly be empty, there exists a box B such that all the points in P are contained in B, and all the points in Q are outside B?

15. Let s_1, s_2, s_3 be the three roots of $x^3 + x^2 + \frac{9}{2}x + 9$.

$$\prod_{i=1}^{3}(4s_i^4 + 81)$$

can be written as $2^a 3^b 5^c$. Find $a + b + c$.