1. There are 80 students in a class. Among them, 40 like math, 30 like science, and 20 do not like math or science. Find the number of students in the class that like both math and science.

   Answer: 10

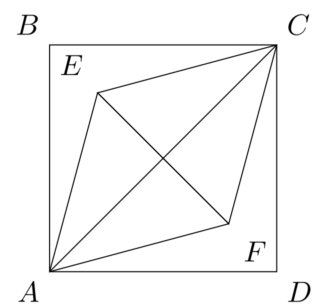
   Solution:

$$40 + 30 + 20 - 80 = \boxed{10}$$

2. Let $ABCD$ be a square. Points E and F are placed inside $ABCD$ so that triangles $\triangle AEF$ and $\triangle CEF$ are equilateral. Compute the ratio of the area of $ABCD$ to the area of $\triangle AEF$.

   Answer: $2\sqrt{3}$

   Solution:



Let s be the side length of square $ABCD$. First, we see that $\triangle AEF$ and $\triangle CEF$ are congruent and have the same side length EF. The height of both triangles is equal to half of AC, the diagonal of the square, which has length $AC = s\sqrt{2}$. The height of an equilateral triangle is equal to $\frac{\sqrt{3}}{2}$ times its side length, so the height of $\triangle AEF$ is $\frac{\sqrt{3}}{2}EF$.

Setting half the length of the square's diagonal equal to the height, we get that $\frac{s\sqrt{2}}{2} = \frac{\sqrt{3}}{2}EF$, or $\frac{1}{2}s^2 = \frac{3}{4}EF^2$. The area of $\triangle AEF$ is equal to $\frac{1}{2} \cdot EF \cdot \frac{\sqrt{3}}{2}EF = \frac{\sqrt{3}}{4}EF^2$. Using our equation from earlier, we have the area of $\triangle AEF$ as $\frac{1}{2\sqrt{3}}s^2$, which gives the ratio of the area of $ABCD$ to the area of $\triangle AEF$ as $\frac{s^2}{\frac{1}{2\sqrt{3}}s^2} = \boxed{2\sqrt{3}}$.

3. How many positive integers $n \leq 2025$ can be written as $n = (p+q)^2 + p + q$, where p and q are (not necessarily distinct) prime divisors of n?

   Answer: 9

   Solution: Let $a|b$ mean that a is a divisor of b. We observe that n only depends on the value of $p+q$, not the values of p or q independently, so we only need to count the possible values of $p+q$. First, note that if we set $p = q$, we have that $n = (2p)^2 + 2p = 4p^2 + 2p$, so $p, q|n$. Now, we assume $p < q$ (since the variables are symmetric). Then, we have $p|(p+q)^2 + p + q$ which is true if and only if $p|q(q+1)$. By symmetry, we also require $q|p(p+1)$. Since $p \neq q$ and p, q are prime, it follows that p and q are coprime and we can conclude that $p|(q+1)$ and $q|(p+1)$. This implies $q - 1 \leq p \leq q + 1$, or $|p - q| = 1$. The only two primes that satisfy this are $2, 3$.

   Now, we count all possible values of $p+q$. There is $2 + 3 = 5$, and then for any prime p we set $q = p$, so we can take any number that is twice a prime. The maximum possible value of $p+q$ is 44, as $(p+q)^2 < (p+q)^2 + p + q < (p+q+1)^2$, and $2025 = 45^2$. We then count the values of $p+q$ as $4, 5, 6, 10, 14, 22, 26, 34, 38$ giving us a total of $\boxed{9}$ possible values of n.