BmMT 2025 Online

Individual Round

June 7, 2025

Time limit: 60 minutes.

Instructions: This test contains 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise.

No calculators. Protractors, rulers, and compasses are permitted.

- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2} + \frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.
- All answers should be written in the form of numbers only. When encountering ratios, express them using integers or fractions and do not use colon notation, for example, use $\frac{3}{4}$ instead of 3:4.
- Do not include any units in your answers.
- For fractions, both improper fractions and mixed numbers are acceptable.

1. Timothe is reserving 100 classrooms for BmMT this year, and each room has exactly one purpose: testing, grading, or activities. Half of the rooms are for testing and four rooms are for grading. How many rooms are left for activities?

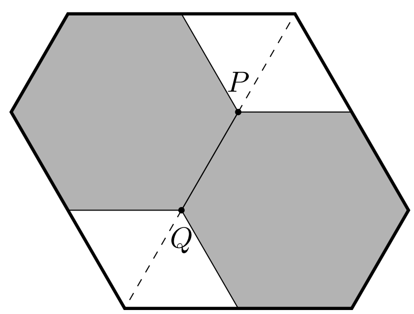
2. Points A, B, and C lie on a straight line, not necessarily in that order. The distance between A and B is 15, and the distance between A and C is 7. What is the sum of all distinct possible distances between B and C?

3. Benji writes down $1, 2, 3, 4, 5$ and Kiran writes down $6, 7, 8, 9, 10$. Later, Kiran erases some of his numbers so that the sum of his remaining numbers equals the sum of Benji's numbers. What is the sum of the numbers that Kiran erased?

4. A bag contains two red marbles, two blue marbles, two green marbles, and two yellow marbles. Richard removes marbles from the bag one at a time without replacement. What is the least number of marbles Richard must remove to guarantee that two of the marbles that he has taken out are the same color?

5. Four consecutive even integers sum to -20. What is the product of these four integers?

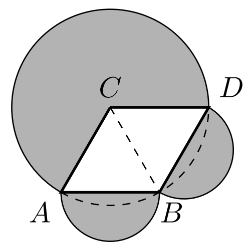
6. Points P and Q are vertices shared by two congruent regular hexagons (shaded) and two equilateral triangles (unshaded) with the same side lengths, as shown below. Together, these shapes form a larger, non-regular hexagon (drawn with a thick border). What is the ratio of the **combined** area of both regular hexagons (the shaded area) to the area of the large hexagon?



7. How many positive factors of 84 are divisible by 3? Note that 1 and 84 are positive factors of 84.

8. Luke the frog lives on a pond with 4 lilypads, labeled 1 through 4. He starts at lilypad 1 and, at every step, hops to a lilypad with a larger number, chosen uniformly at random. Luke continues hopping until he reaches lilypad 4. What is the probability that it takes Luke exactly 2 steps to reach lilypad 4?

9. Around equilateral triangles $\triangle ABC$ and $\triangle BCD$ shown below, a circle centered at C with radius $\overline{CA}$ and semicircles with diameters $\overline{AB}$ and $\overline{BD}$ are drawn. If the total area of the shaded region (the whole shape excluding $\triangle ABC$ and $\triangle BCD$) is 3300π, what is the perimeter of parallelogram $ABCD$?



10. Aaron has four coins. These coins have values of 1, 10, 100, and 1000 cents. Aaron makes a list of all the different numbers of cents he can make with some or all of his coins. If Aaron adds up all the numbers in his list, what is the result **in cents**?

11. Points A, B, and C lie on a circle. Let segment $\overline{BC}$ extend through C to point D such that $\overline{AD}$ is tangent to the circle. If $AC = 11$, $BC = 6$, and $\angle ACD = 90^\circ$, what is CD?

12. Find the sum of all numbers n such that the equation $x^2 - nx + 72 = 0$ has two positive integer solutions, and one solution is an integer multiple of the other.

13. A positive integer n is *two-cool* if the decimal expansion of $n/250$ has exactly one or three digits past the decimal point, excluding trailing zeros. For example, $10/250 = 0.04$ has exactly two digits past the decimal point, and $100/250 = 0.4$ has exactly one digit past the decimal point, so 100 is *two-cool* but 10 is not. How many positive integers less than 35 are *two-cool*?

14. Define a sequence of positive integers $a_1, a_2, a_3, \ldots$ such that $a_1 = 1$ and

$$a_i = 61a_{i-1} + i$$

for all integers $i \geq 2$. What is the smallest positive integer n such that both a_n and a_{n-1} are divisible by 12?

15. Let (a, b, c, d, e) be a permutation of $(1, 3, 5, 7, 9)$. For example, a possible permutation is $(a, b, c, d, e) = (7, 3, 9, 1, 5)$. What is the maximum possible value of $a - ab + abc - abcd + abcde$, where ab, abc, $abcd$, and $abcde$ all represent multiplication (for example, $ab = a \times b$)?

16. Nikki chooses three distinct square cells, A, B, and C, from a 2×6 square grid uniformly at random. What is the probability that square C is contained within the rectangle whose opposite corners are squares A and B? Examples of rectangles with opposite corners at A and B are shown as shaded regions below.

17. Let a, b, and c be positive integers such that $\{a, b, c, 2025 \cdot 6^7\}$ is a geometric sequence that is strictly increasing, meaning $a < b < c < 2025 \cdot 6^7$. Find the number of distinct possible values of a.

18. Let $ABCDEF$ be a regular hexagon. Let P be a point on segment $\overline{BF}$, and let line $\overleftrightarrow{CP}$ intersect segment $\overline{AF}$ at Q. If $AB = 20$ and $\frac{BP}{PF} = \frac{5}{2}$, what is PQ?

19. Danielle calculates the ones digit of each of the 2048 integers $1^{1!}, 2^{2!}, 3^{3!}, \cdots, 2048^{2048!}$. If Danielle adds up all these ones digits, what is the resulting value?

20. There exist strictly increasing arithmetic sequences of real numbers, $\{a, b, c\}$ and $\{p, q, r\}$, having the properties that q is a positive integer greater than 1 and that the equation $x^3 - ax^2 + bx - c = 0$ has solutions p, q, and r. Over all such pairs of increasing arithmetic sequences, what is the least possible value of the product pqr?