## Introduction

Welcome to Cross-Number! This companion document will be available to you throughout the entire duration of the round. It's highly recommended that everyone on your team work through all provided examples.

A reminder about difficulty: Puzzles are arranged in increasing difficulty by section only. Feel free to skip to different sections, where puzzles may be easier.

## Basic Cross-Number

The main goal of a Cross-Number is to use a list of provided numbers to fill in a crossword-like puzzle. A cross-number consists of two components: A grid of squares, and a list of numbers. Place one digit ( $0-9$ ) in every square in the grid, such that:

Every number listed can be found in the grid as a sequence of horizontally-adjacent digits read left to right, or as a sequence of vertically-adjacent digits read top to bottom, with no additional digits before or after the number (in other words, the first and last digit of every number must be on the edge of the grid).

Every sequence of two or more horizontally-adjacent digits and every sequence of two or more vertically-adjacent digits can be read left-to-right or top-to-bottom as a number listed.

Every number two digits or longer is given: this means no other numbers of length 2 or more may appear in the across or down orientation.
Example. Given the numbers 2718, 12345, 31415, fill out the following grid:


## Compact Cross-Number

The rules for Compact Cross-Numbers are the same as for the basic rules. The name difference is to highlight the different shape and higher amount of intersections of these puzzles.

## Blocks Cross-Number

Basic Cross-Numbers omit one rule that will be standard to the rest of the puzzle types.
In Blocks Cross-Numbers, some squares may be left unfilled by any digit. These squares are instead marked as blocks, which is done by coloring in the square completely.

After placing all blocks, the remaining squares in the grid must be connected. These remaining squares must form a valid cross-number as per the Basic rules.

As before, every number listed must be in the grid as a sequence of horizontally-adjacent digits or as a sequence of vertically-adjacent digits. Every sequence of two or more horizontally-adjacent digits and every sequence of two or more vertically-adjacent digits is listed as a number, so no additional numbers may be present in the grid.

For example, two numbers may occupy the same row or column given that they are separated by one or more blocks.

Example. Given the numbers 80, 123, 147, 369, 789, fill out the following grid:


## Variants

Note that each variant is independent of the other variants; additional rules in the Minesweeper variant do not apply to the Reversible variant, for example.

## Variant 1: Minesweeper

This is a variation of the Blocks Cross-Number.
Every digit except 9 must be surrounded by an appropriate number of blocks. The number of blocks neighboring (by neighboring, we mean touching side-to-side or diagonally) a square must equal the digit inside the square. The outside of a grid does not count as a block.

If the digit in a square is 9 , then any number of neighboring squares may be a block.
As before, the remaining squares after placing all blocks must be connected.
Example. Given the numbers 10, 909, 2919, 4299, 7999, fill out the following grid:


## Variant 2: Reversible

In addition to numbers appearing as a sequence of digits from left-to-right or top-to-bottom, numbers can also appear in the grid reading right-to-left or bottom-to-top. For every sequence of two or more horizontally-adjacent digits, either the number read left-to-right or the number read right-to-left is listed as a number. For every sequence of two or more vertically-adjacent digits, either the number read top-to-bottom or the the number read bottom-to-top is listed as a number.

Example. Given the numbers 123, 345, 2105, 5232, fill out the following grid:


Note: in this instance we fill in the numbers 2105 and 5232 in reverse order. Reminder: Leading zeroes ARE permitted, so a number like 120 could appear reversed in a puzzle grid.

## Variant 3: Clues

This section uses the same rules as the Blocks Cross-Number, but the numbers are presented differently.

Instead of providing the exact number, a clue relating to the number will be given. There are some additional pieces of information relating to the position, length, and orientation of the number in the puzzle that may or may not be given.

Some letters are provided on the grid in small text. If a clue begins with a letter (with a colon following it), the leading digit of the corresponding number will be placed in the square marked with that letter. If a clue does not have this information, its corresponding number may be placed anywhere in the grid.

Some clues will specify whether the number is to go across or down. If provided, this information will be at the start of the clue before a colon. Numbers with a clue marked "Across" must be placed in horizontally-adjacent cells and read left-to-right, while numbers with a clue marked "Down" must be placed in vertically-adjacent cells and read top-to-bottom. If not provided, the number may go in either orientation.

Some clues will specify the number of digits of the number they refer to, denoted by a number in parentheses at the end of the clue. This number of digits includes leading zeroes. If this length is not given, the number may be of any length longer than one digit.

Each number two digits or longer in the grid will have a corresponding clue, and no other numbers may appear. As a final reminder, leading zeroes ARE permitted.

For example, consider the following clues and the following grid:


```
A across: A perfect 4th power (3)
C across: The sum of its digits is divisible by }
D across: A multiple of 25
A: A positive multiple of 101
B down: A number with consecutive digits in ascending order (3)
```


## Clue Clarifications

Here, we provide some clarifications on some terminology clues may use. Please read this carefully, as misinterpreting a clue makes solving the puzzles correctly impossible. If you are unsure how to interpret a clue during the round, please refer to this clarification section.

Digits refer to parts of numbers, whereas squares refer to parts of the grid. All squares are either filled with a digit or marked as a block. Blocks are introduced in the Blocks variant.

A palindrome refers to the digits of the number in the grid, not the value of the number itself. For example, 020 would be an example of a palindrome in the grid, even though 20 is not a palindrome.

Adjacent digits of a number are ones that are next to each other when the number is written down. Consecutive digits are ones that differ by 1 from each adjacent digit, and are in increasing or decreasing order. Digits forming a consecutive set are ones that could be rearranged to form consecutive digits.

For example, in the number 5836, 5 and 8 are adjacent but not consecutive. On the other hand, in the number 512943, all of the digits except 9 form a consecutive set, despite not being adjacent. The numbers 456 and 765 are both examples of numbers with consecutive digits.

A semiprime is a number that is the product of exactly two prime numbers. For example, $15=3 \cdot 5$ is a semiprime.

Two numbers are coprime if the only factor they share is 1 . We consider 1 to be coprime with every integer, and 0 to not be coprime with any positive integer except 1 .

Basic
10 points

$$
\begin{array}{|c|}
\hline \text { Basic } 1 \\
\hline 2023 \\
\hline 32768 \\
\hline 65536 \\
52200625 \\
\hline
\end{array}
$$



11 points

| Basic 2 |  |
| ---: | ---: |
| 251 | 252 |
| 353 | 354 |
| 12121 | 451 |
| 24232513232 |  |



## Compact

12 points


12 points
Compact 2

| 10053 | 20469 |
| :---: | :---: |
| 23083 | 34262 |
| 40879 | 43149 |
| 48460 | 65608 |
| 97608 | 99836 |



## Blocks

11 points


12 points

| Blocks 2 |
| :---: |
| 11111 11111 |
| 1111121112 |
| 111112 |
| 2111111 |
| 2111111 |



15 points

## Blocks 3

| 1130411 | 2230422 |
| ---: | ---: |
| 2233311 | 2244411 |
| 11444055522 |  |
| 33666044411 |  |
| 44111011166 |  |
| 55222022244 |  |
| 711555044433 |  |
| 722444066611 |  |
| 744222011166 |  |
| 755222011144 |  |



## Minesweeper

11 points


| 10 | $21 \quad 90$ |
| :---: | :---: |
| 229 | 912 |
| 119921 |  |



12 points


15 points


## Reversible

11 points
Reversible 1


13 points


14 points

| Reversible 3 |  |  |
| :---: | :---: | :---: |
| 12 | 12 | 12 |
| 14 | 15 | 15 |
| 16 | 17 | 18 |
| 021 | 19 |  |
| 024 | 022 | 023 |
| 028 | 025 | 026 |
| 0214 | 218 |  |
|  | 1812 |  |
| 02720 | 62029 |  |
| 1333211135554 |  |  |
| 1666377759992 |  |  |
| 1666544434442 |  |  |
| 1999377728884 |  |  |
| 2888577718883 |  |  |
| 2999122252223 |  |  |
| 3555133321114 |  |  |
| 3555244416664 |  |  |



## Clues

## 11 points

## Clues 1



A Across: A number with only even digits, in strictly descending order (3)
D Across: A number not divisible by 9 (3)
E Across: A number divisible by 11 (2)
A Down: A number with consecutive digits, in ascending order (3)
B Down: A number where the product of the lesser-valued two digits
is equal to the largest digit (3)
C Down: A prime number greater than 10 (2)

## 14 points

Clues 2


A Across: This palindrome is a multiple of 4
G Across: This odd number's digits are in strictly descending order
H Across: This number has exactly two distinct digits (4)
I Across: This number starts and ends with the same digit (4)
J Across: This number is the product of B Down's digits, and its leading digit is 2
A Down: This number only has even digits
B Down: The product of this number's digits has exactly 4 distinct prime factors
C Down: Each digit of this number is 2 more than some digit of D Down
D Down: Each digit of this number is 2 less than some digit of C Down
E Down: This number is less than 10, and the number of blocks in the grid is half of this number. (2)
F Down: Its digits are in strictly ascending order, and 4 of these digits do not appear in any other number in the grid (5)

## 16 points

## Clues 3



A Down: The only number in this grid with every digit 0-9
B Down: The only number in this grid with repeated digits, with all 3 distinct odd digits placed above all 7 even digits
C Down: Adjacent pairs of digits of this number are not consecutive (7)
D Down: A perfect 8th power of a prime number (7)
E Down: The only number greater than 100 with only even digits (4)
F Down: A number with only prime digits (4)
G Down: 4 more than $H$ Down, and the only number in this grid with a unique length (number of digits)
H Down: The only number in this grid that shares a prime factor with every other number in the grid (2)
I Down: An even number, and the only number in this grid filling squares that aren't filled by any other number (6)
Across: A factorial (2)
Across: A number whose digits don't appear in either the number immediately above it, or the number immediately below it (4)
Across: A number with only composite digits, with the first three digits forming a decreasing geometric sequence (4)
Across: A semiprime within 16 of both the number immediately above it, and the number immediately below it (2)
Across: An even number with three consecutive digits in ascending order, the smallest of which is larger than this number's length (6)
Across: The last four digits of this even number are present in reverse order in I Down, one of which is 5 (9)
Across: The only number in the grid greater than 100 with digits in descending order (4)
Across: The only number in the grid with a leading zero, and all of its nonzero digits form a consecutive set (7)
Across: The sum of the digits of the number immediately above it in the grid (2)

Across: This number ending in 2 has one pair of adjacent digits that are not coprime (9)

