Time limit: 10 minutes.
Instructions: For this test, you work in teams of five to solve 50 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

True or False? For questions 1 to 10, write "True" or "False" on your answer sheet.

1. If an integer is divisible by 9, then it is divisible by 3.
2. There exists a real number x that does not satisfy the following inequality: $|8-4x| \geq -2$.
3. If $ab = ac$ and a, b, c are integers, then b must equal c.
4. If a, b, c are positive integers, then it is always true that $||a-b|-c| = |a-|b-c||$.
5. If a right triangle has a side of length a and a side of length b, then its area is always $\frac{1}{2}ab$.
6. There exist integers (a, b, c, n) such that $a^n + b^n = c^n$.
7. The probability of obtaining an even number of heads is less than the probability of obtaining an odd number of heads when flipping 5 coins.
8. 2020 has exactly 4 positive factors that are 1 more than a perfect square.
9. There exist 10 distinct non-negative integers that sum to 43.
10. 2^{100} is more than 30 digits long when written in base 10.

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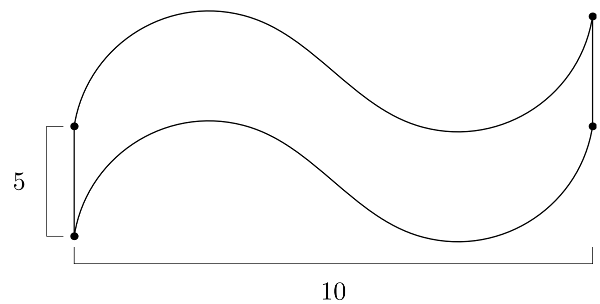
11. How many positive prime numbers are less than 3?
12. What is the perimeter of a regular octagon with side length 5?
13. Compute $(5-2)^2 \times 3 - 6$.
14. If $x = y + z$ and $x + y = z$, compute the value of y.
15. How many positive factors does 205 have?
16. A school with 8 classes is going on a field trip. Each class has 20 students and 1 teacher. What is the fewest number of 40-passenger buses needed to transport the 8 classes?
17. Compute the sum of the first 10 positive integer multiples of 3.
18. What is the area of a trapezoid with side of length 5, 10, 5, and 18 if the sides of length 10 and 18 are parallel?
19. The measure of the largest angle in triangle ABC is 132 degrees. Let EFG be a triangle such that $EF = 2AB$, $FG = 2BC$, and $GE = 2CA$. What is the degree measure of the largest angle of triangle EFG?

20. What is the area of the S-shaped figure below, which has constant vertical height 5 and width 10?



21. Find the integer closest to $4 \cdot \sqrt{14}$.

22. A computer is priced at 1000 dollars. Jillian has a 25 percent off discount and a 50 dollar off discount, and can use both discounts in either order. What is the lowest price in dollars that Jillian can pay for the computer?

23. Geometric sequences $\{a_n\}$ and $\{b_n\}$ have the same common ratio. If $a_1 = 2b_1$, compute $\frac{a_{20}}{b_{20}}$.

24. Two consecutive perfect squares differ by 15. What is the positive square root of the larger of the two perfect squares?

25. Joey and Austin each individually pick an integer from 0 to 9, inclusive. Joey wins if the product of the two numbers is 0, and Austin wins otherwise. Assuming both players play optimally, what is the probability Joey wins?

26. What is the minimum number of coins needed to make 87 cents using pennies, nickels, dimes, and quarters? A penny is 1 cent, a nickel is 5 cents, a dime is 10 cents, and a quarter is 25 cents.

27. A rectangle of height 10 and width 24 is inscribed in a circle. What is the circumference of that circle? Express your answer in terms of π.

28. Three teams each with three people decide to shake hands. If everyone does not shake his or her teammates' hands, but shakes hands with everyone else exactly once, how many handshakes occur?

29. Dylan takes marbles from a bag with 2 red marbles, 2 green marbles, 3 blue marbles, and 4 purple marbles. If Dylan takes marbles out of the bag without replacement, and does not look at the marbles he takes, how many marbles must Dylan take to ensure that he has at least three different colors of marbles?

30. What's the maximum number of circles of radius 4 that fit into a 24×15 rectangle without overlap?

31. What is the surface area of a rectangular prism with volume 30 if it has a square base and all edge lengths are integers?

32. John received the scores of 80, 87, 91, 93, and 100 on his past 5 tests. If John can only receive integer scores between 0 and 100 inclusive, how many ordered pairs of scores (a, b) are there for his last two tests such that the average of his seven test scores is exactly 93?

33. Tim is currently three times as old as Tom. In 10 years, Tim will be twice as old as Tom. How old is Tom currently?

34. Thirty people are equally spaced around a circular table and are numbered 1 through 30 going counterclockwise. What is the degree measure of the angle formed by the people numbered 1, 15, and 4 where the vertex of the angle is at the person numbered 15?

35. What is the smallest positive integer that is divisible by 44 and has only ones and zeros in its base-ten representation?

36. At BMT 2018, Eric is taking two focus (individual) rounds, each of which consists of 5 questions. In how many different ways can he get exactly 5 of the 10 total problems correct among both tests, up to the specific problems he gets right and wrong on each test? (For example, if Eric gets the first four correct on the first test and only the first question on the second test, this is different from getting the last four correct on the first test and only the first question on the second test).

37. Phil's eighteen-wheeler truck has eighteen tires and six spare tires. Each tire has a circumference of 2 meters. Phil needs to make a 4800-meter trip and wants to make sure all 24 of his tires spin the same number of times. How many times will each tire spin?

38. How many positive two-digit integers are 45 more than their reverse? For example, 90 is 81 more than its reverse 09.

39. In a square of area 1, what is the area of the region formed by the set of all points inside the square that are closer to the center of the square than to any of its four vertices?

40. Compute the sum of all irreducible fractions of the form $\frac{a}{34}$, where a is an integer between 1 and 33, inclusive.

41. An ant is at vertex A of cube $ABCD-EFGH$, where $ABCD$ is one of the square faces with AC as a diagonal. Each second, the ant randomly chooses a vertex adjacent to its current vertex to crawl towards. What is the probability the ant is on vertex C after 9 seconds?

42. What is the sum of all integers k such that $\frac{2020^3}{k}$ is an integer?

43. Out of the eight interior angles of a convex octagon (an octagon where none of the interior angles are greater than $180°$), at most how many of them can be $120°$?

44. $P(x)$ and $Q(x)$ are two quadratic functions such that taking the difference between them results in a quadratic function with no zeroes. When graphed in the xy-plane, at how many points do P and Q intersect?

45. Alice and Bob each roll a six-sided die, obtaining two (not necessarily distinct) numbers A and B. What is the probability that both $\underline{A}\,\underline{B}$ and $\underline{B}\,\underline{A}$ are prime (where $\underline{X}\,\underline{Y}$ is the two digit number with ones digit Y and tens digit X for $1 \le X \le 9$, $0 \le Y \le 9$)?

46. What is the radius of a circle inscribed in a triangle with side lengths 6, 8, and 10?

47. Freya flips a coin 10 times. Compute the probability that she never flips a head immediately after flipping a tail.

48. How many ordered pairs of integers (x, y) satisfy $x^2 + y^2 = 2^7$?

49. A cube is painted black on all faces then cut into 1000 smaller congruent cubes. One of these smaller cubes is selected at random and rolled like a die. What is the probability that the top face is black?

50. Given that

$$\frac{2^{2018}+1}{2^{2019}} = x$$

and

$$x^2 - x = -\frac{1}{4} + c,$$

compute the value of c.