Contestant 1

A. Suppose:

$$x \star y = xy - 2x - 3y$$

Let A be given by:

$$A = (12 \star 4) \star (4 \star 12)$$

Find A.

Y. A number is *cool* if it is divisible by 5 and has only odd digits. An *awesome* number is a *cool* number that has X digits. If there are Y *awesome* numbers, find Y.

C. There is a triangular field of grass, bounded by an iron fence. The field has side lengths $6, B, 10$ miles. There are three hungry cows, each tied to one of the vertices of the field. The length of the ropes that tie them is 2 mile. Suppose they eat all the grass they can within their reach. Then let C denote the area of the field covered by the remaining grass. Find C.

Contestant 2

X. In the cryptarithm $RED + SOX = SLOW$, each letter represents a distinct digit in base 10, and $D + L = 7$. If $W = 2$, find X.

B. Bugs Bunny is running a 12 mile race. He runs the first 6 miles with speed 6 mph. And he runs the next 6 miles with speed A. If his average speed is B, find B.

Z. In a class with Y students, desks are arranged in a $\sqrt{Y} \times \sqrt{Y}$ grid. Of course, with such a large class, there are always a fair amount of students missing. One day, the teacher realized that each of the present students had *at least* one absent neighbor in the (up to) 8 adjacent seats. What is the maximum possible number of students who could have attended class that day? Denote this by Z.

Contestant 1

A. A line l is drawn through a square such that it splits the square into two regions, each with area 32. If the line intersects the square 1 unit from one vertex, let A be the length of the portion of l contained entirely within the square. Find A.

Y. The *natural* characteristic of a function is defined to be the amount of natural numbers that are not in the range of the function, when its domain is taken to be the natural numbers. For example, the function $f(x, y, z) = 3x + 4y + 5z$ has a *natural* characteristic of 2, since 1 and 2 are not in its range, but every other positive integer is. Find the number Y such that the function $f(x, y) = Xx + Yy$ has a *natural* characteristic of 7.

C. Consider the set S of all four-digit palindromes q such that $q \cdot B$ is an integer. Find the remainder when the sum of the numbers in S is divided by 1001. Denote this by C.

Contestant 2

X. An orange box contains 12 red balls and 18 yellow balls. A purple box contains 20 blue balls and 16 red balls. A green box contains X yellow balls and 8 blue balls. If the ratio of yellow to other balls in the green box is equal to the ratio of yellow to other balls overall, find X.

B. Ophelia has A dice, each with sides numbered 1 to 6, and she wants to roll them all at once. Let B be the probability that the sum of her A results is divisible by 6. Find B.

Z. If $g(x) = x^3 - 1$ and $h(x) = x^2 - 4x - 3$, find the value of Z such that $g(Z) + h(Z) = Y!$.

Contestant 1

A. Consider the following statements made by five different people.

(a) Person b and Person c are lying!
(b) Person c and Person d are lying!
(c) Person d and Person e are lying!
(d) I am telling the truth!
(e) Person a and Person b are not both lying!

What is the maximum possible number of statements that could be true? Denote this by A.

Y. Jim has 3 quarters, 4 nickels, 5 dimes, and X pennies. In how many ways can he make 97 cents? Denote this by Y.

C. Circle O_2 of radius 2 is internally tangent to circle O_1 of radius B. A radius of circle O_1 is drawn tangent to circle O_2, and the radius of circle O_2 perpendicular to the tangent is extended until it intersects circle O_1 at P. Find the distance from P to the center of circle O_2. Denote this by C.

Contestant 2

X. Let $f(x) = 2x^2 - 13x + 7$. There exists one point on $f(x)$ in Quadrant I (positive x, y axes) such that the x-coordinate is 5 greater than the y-coordinate. Find the x-coordinate X.

B. In the world of Samdep, the order of operations is the reverse of what it is here; that is, addition and subtraction come before multiplication and division, and exponents are evaluated last. Then, if $A + 3 \,/\, 6 - 4 \cdot 3 \,\hat{}\, 2 = B \cdot 4 + 5$, find B.

Z. The Fibonacci sequence $F_n = F_{n-1} + F_{n-2}$ is defined such that $F_0 = 0$ and $F_1 = 1$. The remainder when F_k is divided by Y is 1. If $0 < k \leq 9000$, find the number of possible values for k. Denote this by Z.

Contestant 1

A. A nondegenerate quadrilateral has 1 side of length 1 and 2 sides of length 2. If the fourth side also has an integer length, how many possibilities are there for it? Denote this by A.

Y. Everyone knows the Bay Area is best! Since $B \cdot A \cdot Y$ is best, it has precisely 4 factors in the "teens" ($13 \leq n \leq 19$). Find the smallest possible value of Y.

C. Right triangle RST has hypotenuse ST and point D somewhere along side ST such that RD has length A. Then, draw DE and DF such that E and F are on RS, DE bisects $\angle RDS$, and $DF = RF = SF$. If RF also has length A, then find $\angle EDF$. Denote its measure by C.

Contestant 2

X. Let X be the area of a triangle with sides A, B, and C. Find X.

B. Let S represent the set of all integer degree measures ($0^\circ < \theta \leq 360^\circ$) that are divisible by C but not A. Find the number of angles in S larger than a right angle. Denote this by B.

Z. The point (X, Y, Z) is a lattice point (integer coordinates) that is the center of the sphere $x^2 + y^2 + z^2 + \alpha x + \beta y + \gamma z + 35000 = 0$. If the radius of the sphere is 1, find Z.